FLYING QUALITIES CONSTRAINTS IN THE MULTIDISCIPLINARY DESIGN OPTIMIZATION OF A SUPERSONIC TAILLESS LONG RANGE AIRCRAFT

Craig C. Morris,
Graduate Research Assistant

Joseph A. Schetz,
Co-Advisor, Holder of Frederick D. Durham Chair

and

Cornel Sultan,
Co-Advisor, Assistant Professor

Department of Aerospace and Ocean Engineering
Virginia Tech, Blacksburg, VA, 24061, USA

The increased complexity of next generation aircraft has popularized Multidisciplinary Design Optimization (MDO) as an aircraft design tool. However, MDO efforts consistently neglect aircraft stability and control analyses beyond simple static assessments. Not only does this limit the potential of the MDO to identify an optimal design, but it also fails to assess the MIL-STD or FAR handling qualities requirements the aircraft must achieve. In an effort to bring aircraft handling qualities and dynamic performance into the MDO of a tailless supersonic aircraft, a dynamic stability assessment tool is under development with the goal of automating the feedback control system design to facilitate optimization of the aircraft control effector configuration. Convex optimization tools and a linear matrix inequality problem formulation make the problem computationally tractable for use in MDO. Model and framework development are presented here, along with the results of a demonstration/validation case on a more conventional subsonic transport aircraft. Future research efforts are also discussed, and focus on a covariance constrained control technique that is under development which utilizes a novel translation of modal parameter guidelines to state variance upper bounds for a given initial condition.

I. Introduction

Since before the Wright Flyer left the ground, aircraft designers have struggled with the necessary and challenging task of maintaining stability and control of the vehicle. In 1809, George Cayley became the first to recognize the force balance needed for flight in the first part of his three-part treatise on aerodynamics entitled, “On Aerial Navigation.” Through observations of nature and his experiments with gliders, Cayley soon after discovered the need for wing dihedral and center of gravity positioning as crucial elements for stability in an aircraft’s design. Even in the 21st century, stability and control continues to pose a challenge to aircraft designers. The traditional design process all too often delays thorough study of the aircraft’s stability and control until after the conceptual design process. While control integration efforts have been made on specific configurations (so-called “control-configured aircraft”), there are many examples of modern aircraft which suffer from control design limitations that went unnoticed until late in the design process. This strategy of aircraft design is not only costly, as late redesigns are expensive, but also performance limiting.

At the conceptual design level, traditional aircraft stability and control design consists of little more than volume coefficients and static analyses. This approach proved successful in the design of tube-and-wing aircraft with reasonably stiff structures and lightly coupled wing/tail/body aerodynamics. It wasn’t until the preliminary design stage was
reached that performance analyses involving particular control effectors and their sizes was accomplished, and even then aircraft dynamics were often ignored. Static analyses are necessary to properly characterize the flight dynamics of an aircraft, yet they are far from sufficient. In the case of a tailless supersonic aircraft, where the physics are highly coupled about all three axes, the dynamics can not be ignored.

The work that follows outlines efforts to include dynamic stability assessment into an aircraft Multi-disciplinary Design Optimization (MDO) framework for the conceptual design of a next generation supersonic tailless aircraft—also called Efficient Supersonic Air Vehicles (ESAV). The MDO has been developed through the Collaborative Center for Multidisciplinary Sciences at Virginia Tech (VT), along with its partners at the Air Force Research Laboratory and Wright State University. In recent years, MDO has emerged as an effective tool for the intelligent design of complex systems, including the next generation of aircraft. Previous efforts by both academia and government agencies have sought to address the shortcomings of conceptual design stability and control through MDO, but the complexity of developing a truly general tool prevents their reuse here.

A supersonic tailless aircraft requires retention of additional coupling terms in the equations of motion, and the use of nontraditional control effectors prevents the use of database techniques as a valid means of aerodynamic analysis. That being said, the transport aircraft demonstration case used herein makes use of both decoupled equations of motion and semi-empirical databases to speed development and testing of the dynamic assessment tools.

The existing framework is composed of a series of analysis modules, assembled in a particular sequence to facilitate optimization and reduce any backwards information flow. This type of structure is known as an $N^2$ diagram, and an example of the VT ESAV $N^2$ diagram can be found in Figure 2. Information management and program execution are handled through Phoenix Integration’s ModelCenter software.

The framework is well described in a publication by Allison, et al., and includes a validation study of the results using the B-58 supersonic bomber. Many of the analyses are developed in-house, including the geometry model, engine exhaust wash surfaces analysis, and the stability and control module. Third party software packages utilized include the Numerical Propulsion Simulation System (NPSS), Flight Optimization System (FLOPS), and ZONAIR—a medium fidelity aerodynamic solver.

III. Stability and Control in MDO

While MDO is frequently championed as a means to address the shortcomings of the typical design process, very few aircraft MDO efforts include a stability and control analysis. Those that do address stability and control typically rely on static sizing scenarios like takeoff nose-up moment generation and cruise trim to size their effectors. As mentioned, only a few efforts have sought to tackle the dynamic performance problem, and their extension to the ESAV optimization is limited by assumptions made in their development. The goal for the VT ESAV MDO stability and control module is to address the known roll/yaw control issues that plague tailless, slender aircraft through the inclusion of the coupled six degree of freedom equations of motion in the dynamics model.

In the VT ESAV MDO framework, the control of the geometric design variables, including control effectors, is maintained globally—meaning for a given iteration down the $N^2$ diagram, the stability and control module is responsible for an analysis rather than a design (the configuration is frozen for each iteration). As the focus of this paper is the stability and control analysis, it is important to provide a brief explanation of information flow in the $N^2$ diagram.
that precedes the stability and control module. A static and dynamic analysis of the aircraft will require knowledge of the mass properties, geometry, aerodynamics, and propulsion (should thrust be included as an effector or state). First, the propulsion and geometry analyses perform an iteration to construct engine inlets and finalize the configuration geometry before passing it on to aerodynamics. The aerodynamics module takes the configuration geometry and an off-line constructed input file dictating the desired flight conditions and produces an aerodynamic performance database. Information from this database is passed on to the structures module for loads analysis and stability and control for performance information. Finally, an internal structural optimization and a code for prediction of secondary aircraft weights are used to estimate the aircraft gross weights and inertias. Thus, a significant amount of information is passed downstream to the stability and control module all at once, necessitating a framework for data storage and to support stability and control analyses.

A. Matlab Object Oriented Stability and Control Framework

To facilitate the addition of a stability and control analysis into the MDO, a Matlab Object Oriented Stability and Control Framework (MOOSCF) was created. This framework, as outlined by the block diagram in Figure 3, creates a plane object with all necessary properties stored as private attributes. Flight condition dependent properties are also stored as object attributes, and each flight condition possesses further attributes describing aerodynamic and inertial properties under those conditions. This data is passed in from the aerodynamics and structural models that are upstream in the $N^2$ diagram. A framework of this type is beneficial in that it standardizes the format of the data, allowing for only one set of tools to be developed to analyze the stability and control characteristics of the aircraft. As the optimizer in the MDO varies the aircraft geometry and properties, only the attributes need be changed. This is a significant computational time savings when the constructor and destructor of the plane object do not have to be called with each iteration. Data from the framework can then be accessed by both the static and the dynamic analysis routines, with the results passed back into the framework to be stored as aircraft attributes.

B. Static Stability and Control

Static stability describes an aircraft’s initial behavior when disturbed from an equilibrium, or trimmed, position. It is little more than summing the forces and
moments on the plane when perturbed some small amount from an equilibrium, and it is widely considered to be an excellent indicator of an aircraft’s overall stability. An aircraft that is too statically stable will feel sluggish and heavy, while a marginally stable aircraft will be very sensitive to even subtle disturbances. In addition, the static stability of an aircraft has first-order implications on trim drag (see Figure 22.2 in Nicolai{17}) and control power required. No amount of control system design can overcome the inability of an aircraft to generate static forces and moments, thus the study of static stability can not be ignored.

The first step in constructing a static stability analysis for the VT ESAV MDO was identifying critical flight regimes. Past supersonic aircraft design efforts found the takeoff nose-up moment generation scenario to be a critical control surface sizing condition, as well as the crosswind landing requirement.\textsuperscript{14} In addition, a subsonic cruise trim, supersonic cruise trim, transonic push trim, and a 2.5 $g$ turning maneuver were selected here as critical cases. Many more critical flight conditions have been identified by others,\textsuperscript{10, 18} and the simplicity of the MOOSCF allows for their rapid addition without significant framework redesign. These cases act as a go/no-go gauge for the MDO, feeding back information to the optimizer to identify the undersized effectors and re-iterate the design.

C. Dynamic Stability and Control

An aircraft that is statically stable is not necessarily dynamically stable.\textsuperscript{19} Static analyses only describe the aircraft’s initial tendency about equilibrium and do not include the time history of the response. Additionally, aircraft designers must address Military Standard (MIL-STD) flying qualities guidelines—a distinctly dynamic characteristic of an aircraft. MIL-STD Level 1 flying qualities requirements prescribe modal parameter constraints, such as natural frequency and damping ratio limits, as a function of the type of aircraft and flight condition. These constraints effectively describe a domain of acceptable closed-loop pole locations like the example shown in Figure 4b. Thus, in order to perform a dynamic analysis, the designer must be interested in the closed-loop system behavior, necessitating the design of the control architecture.

IV. Stability and Control Module Development

Following a thorough literature review, several different approaches to developing the stability and control module for the VT ESAV MDO framework were undertaken. First was the approach detailed at the 2012 Virginia Space Grant Consortium conference, which relied on stochastic gradient based optimization techniques to discover solutions to the inverse optimal control problem which drove the closed-loop aircraft dynamics to be satisfactory.\textsuperscript{20} Though the approach is mathematically valid, the implementation was difficult, and the stochastic algorithms failed to handle the nonlinearities present between design variables and the modal decomposition. This optimal pole placement strategy was later replaced by a linear matrix inequality (LMI) technique which leveraged convex optimization algorithms to develop static state feedback control laws which satisfied the MIL-STD flying qualities guidelines.

In the next section, the LMI approach with Regional Pole Placement constraints is presented in detail. An example problem and results are given in the section following that, including a discussion of the shortcomings and limitations of the method. Finally, in Section V, a novel strategy currently under development for addressing these shortcomings is presented.

A. LMI with Regional Pole Placement

The methods detailed in this section rely heavily upon the work of Kaminer, et al.\textsuperscript{17} The objective is to develop an LMI feasibility statement for the existence of a static state feedback controller subject to constraints on the maximum actuator deflection, $u_{\text{max}}$, the maximum actuator rate, $\dot{u}_{\text{max}}$, and the closed-loop pole locations using methods developed by Chilali and Gahinet.\textsuperscript{21} The full details of the development can be found in the paper by Kaminer, et al.,\textsuperscript{17} and the precedent paper by Niewoehner, et al.\textsuperscript{22} Frequent references are also made to Boyd’s seminal work on linear matrix inequalities and control,\textsuperscript{23} from which many of the fundamental LMI’s are derived. The LMI constraints are not rigorously rederived here, but the necessary equations are included for completeness.

The power of LMI’s has been realized in recent years due to the development of robust convex optimization tools and modern computing. Boyd offers an excellent introduction to LMI’s in his book,\textsuperscript{23} and the newcomer to the subject is encouraged to begin their study there. Since LMI’s are inequality constraints, multiple LMI’s can be appended together into a larger LMI by simple concatenation of matrices along the diagonal, creating a larger matrix. This characteristic was pivotal in the development of Kaminer’s method, as constraints on the actuator deflection, deflection rate, and the closed loop eigenvalues could be developed separately and restated as.
feedback control system design, but it does require an estimation of the actuator bandwidth. Kaminer, et al. use a first order transfer function of the form

$$u = \frac{a}{s + a}, \quad (3)$$

where $a$ is the actuator bandwidth, to model the actuator. This leads to a new system,

$$\mathcal{G} = \begin{cases} \dot{x} = Ax + Bu \\ u = Kx \end{cases} \iff \begin{cases} \dot{x} = (A + BK)x \\ u = Kx \end{cases}, \quad (1)$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^1$, and $A$, $B$, and $K$ are of appropriate dimensions. The selection of $u$ as a scalar in $\mathbb{R}^1$ is not a necessity, but is an artifact of the single input implementation used in the example problem of Section IV-B. Borrowing the notation used by Kaminer et al., the stabilizing constant state feedback controller, $K$, exists if \{ $Y > 0, W \in R^{1 \times n}$ \}, such that the following LMI’s have a solution:

$$\begin{bmatrix} 1 & x(0)^T \\ x(0) & Y \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} Y & W^T \\ W & u_{\text{max}}^2 \end{bmatrix} \geq 0,$$

$$AY + BW + YA^T + BW^T < 0.$$ (2)

If the LMI’s are feasible, $K = WY^{-1}$, and the static state feedback control law thus developed guarantees the stabilization of the system, subject to the initial condition, without exceeding the actuator deflection limit.

2. Maximum Actuator Rate Constraint

The actuator rate can be limited by reformulating the system to include a first-order actuator model, effectively appending the actuator state to the state vector and changing the input to the actuator command. Adding the first-order actuator model to the system adds an element of realism often neglected by feedback control system design, but it does require an estimation of the actuator bandwidth. Kaminer, et al. use a first order transfer function of the form

$$u = \frac{a}{s + a}, \quad (3)$$

where $a$ is the actuator bandwidth, to model the actuator. This leads to a new system,

$$\mathcal{G} = \begin{cases} \dot{x} = Ax + Bu \\ u = K[x^T x_a]^T \end{cases} \iff \begin{cases} \dot{x} = A x + Bu \\ u = K[x^T x_a]^T \end{cases}, \quad (4)$$

$$\dot{x}_a = -a x_a + a u$$

$$\dot{x}_a = a(K - [0 \ 1]) \nu$$

where $\nu = \begin{bmatrix} x \\ x_a \end{bmatrix}$ and the new system matrices are

$$F = \begin{bmatrix} A & 0 \\ 0 & -a \end{bmatrix} \in \mathbb{R}^{(n+1) \times (n+1)}$$

$$G = \begin{bmatrix} B \\ a \end{bmatrix} \in \mathbb{R}^{(n+1) \times 1}.$$ (5)

With the formulation used in Eq. (5), the actuator command can now be regulated in a similar manner to the actuator deflection. This time, however, the bounding operation will be placed on the system output rather than the input. Using Boyd’s output bounding process, an LMI constraint defining a stabilizing controller with actuator rate limits is given by

$$\begin{bmatrix} P \\ a[K - [0 \ I]] \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} x(0)^T P x(0) \leq 1, \quad (6)$$

Finally, this set of equations must be reposed using the relationships $K = WY^{-1}$ and $P = Y^{-1}$ in order to make the third expression linear and be of the same matrix variables as Eq. (2). Carrying out this change of variables and utilizing the Schur complement to
rearrange the second expression, the actuator rate constraint LMI is given as

\[
\begin{bmatrix}
P & a[W - [0 \ I]Y] \\
a[W - [0 \ I]Y] & \frac{1}{u_{\text{max}}^2}
\end{bmatrix} \geq 0,
\]

\[
\begin{bmatrix}
I & x(0)^T \\
x(0)^T & Y
\end{bmatrix} \geq 0,
\]

\[FY + GW + YF^T + GW^T < 0.\]

3. Regional Pole Placement Constraint

The final constraint placed on the feedback control system design is a regional pole placement constraint. First developed by Chilali and Gahinet,\textsuperscript{21} the method produces a static feedback $H_\infty$ controller which places the closed-loop eigenvalues into a convex region in the open left-half plane. Their statement of the region as an LMI constraint preserves the convexity of the $H_\infty$ controller design problem and allows for the use of fast and efficient convex optimization methods. Of particular interest is the so-called $S(\beta, r, \theta)$ region, depicted in Figure 4a, due to its similarity in structure to the MILSPEC flying qualities boundaries in Figure 4b. The region is constructed by intersecting three simple LMI regions of the form

\[AX + XA^T + 2\beta X < 0,\]  \hspace{1cm} (8)

\[-rX \begin{bmatrix} AX \\ XA^T \end{bmatrix} < 0,\]  \hspace{1cm} (9)

\[
\begin{bmatrix}
\sin \theta \left(AX + XA^T\right) & \cos \theta \left(AX - XA^T\right) \\
\cos \theta \left(XA^T - AX\right) & \sin \theta \left(AX + XA^T\right)
\end{bmatrix} < 0,
\]

(10)

where Eqs. (8), (9), and (10) refer to the vertical limit ($\beta$), the radial limit ($r$), and the angular limit ($\theta$), respectively (see Figure 4a). Note that these equations have been posed according to the system described in Eq. (1), and will need to be recast in terms of the augmented system in Eq. (4) to work simultaneously with the previous two actuator constraints. This has already been done by Kaminer, et al.,\textsuperscript{17} though they choose not to use the radial boundary due to the fast mode associated with the control surface—a mode that is likely to violate the radial constraint. The results are reproduced by Morris, et al.,\textsuperscript{24} using the nomenclature found in Eq. 4, but are not repeated here for brevity. A graphical comparison of the MIL-STD and the convex approximation can be seen in Figure 5.

The three constraints developed by Boyd,\textsuperscript{23} Chilali and Gahinet,\textsuperscript{21} and Kaminer, et al.,\textsuperscript{17} can now be combined to make a larger LMI. To summarize, a matrix inequality constraint has been developed to constrain the actuator amplitude, the actuator rate, and the closed loop pole locations. The mathematical statement of the constraints is based on the work of Kaminer, et al.,\textsuperscript{17} and it serves as the foundation for the developmental aircraft problem as well as the path forward to implementation on the ESAV. The MOOSCF supports the construction and analysis of the LMI constraints, and feasible solution searches are accomplished using the built-in LMI solvers in the Matlab Robust Controls Toolbox. It is important to note that the LMI statement of the above constraints is a sufficient condition, and therefore offers a conservative answer to the feasibility problem.
B. Framework Development Aircraft: The Subsonic Transport

In order to hasten framework development, as well as provide some means of validation, a traditional transport aircraft was modeled using the MOOSCF and Digital DATCOM—a semi-empirical aerodynamic and stability and control database. The modeled aircraft is very much like a Boeing 737-800, and possesses several characteristics which are beneficial for method development: decoupling of the longitudinal and lateral/directional dynamics is more easily justified than for the ESAV, the aerodynamic data for traditional elevator control is readily available through DATCOM, and the supersonic flight regime can be eliminated from the flight condition criteria. Figure 6 shows the subsonic transport as modeled in Digital DATCOM.

The objective of the transport aircraft study is to size the horizontal tail subject to both static and dynamic constraints. A fundamental assumption is made in that FAA regulations would allow for the relaxation of static stability (or even instability) of commercial transport aircraft. A combination of static and dynamic constraints are analyzed for the configuration at varying horizontal tail sizes, and a traditional horizontal tail sizing scissor plot is constructed. The scissor plot offers a graphical means of identifying the smallest horizontal tail that is capable of satisfying all of the required constraints. Figure 7 shows the results of this study. On the horizontal axis is the center of gravity (CG) location in percent mean aerodynamic chord (m.a.c). The vertical axis is the horizontal tail area ratio—or the ratio of the tail area to the wing area.

Forward CG limits are determined by static constraints on the takeoff rotation, trim at maximum lift condition, and tail stall recovery at maximum lift condition. The methods of Torenbeek\textsuperscript{25} were used to generate these curves for the transport aircraft. Aft CG limits are traditionally set by minimum static margin constraints. In Figure 7, a static margin of 8% is shown, a value found to be typical of transport aircraft. To make use of the scissor plot, an aircraft designer simply selects the smallest horizontal tail area ratio that satisfies all of the constraints and provides the desired minimum CG range—in this case, an assumed CG range of 20% m.a.c. is selected, which is approximately that of the 737-800. Some range of CG locations must be allotted for, as passenger movement, variations in cargo loading, and fuel consumption all but guarantee a CG that will not remain stationary during flight.

Using only the static constraints, the corresponding minimum tail area ratio is indicated on the plot as approximately 24%. If the minimum static margin constraint can be ignored (i.e. through relaxation of FAA requirements), the tail area ratio can be reduced until the first dynamic constraint becomes active. In this example, the horizontal gust at the cruise condition is the limiting case, and the smallest possible tail area ratio for which the feedback control system can still provide Level 1 flying qualities is indicated as approximately 18%. The full details of the analysis, including any additional assumptions and the chosen flight conditions, can be found in the reference by Morris, et al.\textsuperscript{24} For comparison, the resultant horizontal tail design using the static constraints is shown in Figure 8 in black, and the relaxed static stability tail achieved using the dynamic constraints is shown in red. The relaxed static stability tail achieves a 31% reduction in area over the statically designed tail.

While the approach detailed in Section IV-A was
used to generate the subsonic transport results, it suffers from a very significant limitation. The regional pole placement strategy developed by Chilali and Gahinet\textsuperscript{21} effectively reshapes the complex plane to alter the definition of stability from simply being the open left-half plane, to being a convex subspace of the left half-plane. The consequence of this approach is that every eigenvalue of the system is subjected to the same regional pole placement constraints. For the analysis of an aircraft’s longitudinal equations of motion, this is a significant limitation since the short period mode has much more restrictive modal requirements than does the phugoid mode. Furthermore, this problem will grow worse as the dimension of the state space increases, making it impractical for implementation on the coupled aircraft equations of motion.

V. Future Research: Variance Constrained Flying Qualities

Future research efforts are focused on replacing the regional pole placement constraint with a constraint that is more conducive to scaling for larger systems. Currently under development is a novel approach for replacing the closed-loop modal requirements given by the MIL-STD with state variance upper bounds. This Variance Constrained Flying Qualities (VCFQ) approach relies upon a unique relationship between the state variances of a single input linear time invariant system and the coefficients of the closed-loop characteristic polynomial. The relationship was first identified by Skelton, Iwasaki, and Grigoriadis for an
alternative purpose, but by reinterpreting the single input as an impulsive state disturbance, it is possible to translate modal parameter constraints to state variance upper bounds when provided an initial condition. An LMI constraint can then be developed to design the feedback control system subject to state variance upper bounds along with the actuator constraints previously described. Details on the specifics of implementation are currently under development, and are expected to appear in a publication at the AIAA Guidance, Navigation, and Controls Conference in August, 2013.

VI. Conclusion

Aircraft conceptual design has long lacked sufficient stability and control analyses. Late redesigns and control afterthoughts are quite costly and often degrade the aircraft’s performance. Conceptual design dynamic performance assessment seeks to push closed-loop control and effector design earlier into the design process. Some conceptual design MDO efforts in the past have made use of static stability analyses, but the complexity of a tailless supersonic aircraft will require a more thorough dynamic assessment. A well designed dynamic performance analysis tool would have a broader application than the VT ESAV MDO, as MDO efforts of other aircraft would benefit as well.

The work presented represents only a fraction of the challenge of developing a full aircraft MDO. Many other discipline models needed development just to have sufficient information to perform a stability and control analysis. Upgrades to geometry, propulsion, structures, and aerodynamic analyses were needed to generate the necessary data for a dynamic assessment. While time consuming, these efforts to upgrade the MDO also serve to push the computational fidelity to a higher level. The addition of the Matlab object oriented framework was also a substantial upgrade to the MDO, supporting not only the data handling for stability and control, but also acting as the interface to dynamic analyses.

While initial efforts to incorporate the optimal pole placement strategy proved disappointing, the understanding developed during the process was a crucial step towards developing more complex strategies. Linear Matrix Inequalities offer a powerful tool to the control designer, and even more so for an aircraft conceptual designer interested in a preliminary control system analysis. Convex optimization algorithms can provide solutions to LMI formulated problems in fractions of a second, which makes addressing the feasibility statement needed for the VT ESAV MDO framework tractable. The control systems designed are not intended to be installed on any aircraft, or to serve as final design algorithms, but rather to address the question of ‘Can this aircraft configuration fly, and can it do so in a satisfactory manner?’. Conceptual designers have a high level need to know whether a suite of effectors or a particular geometric configuration is capable of achieving mission goals in order to best handle the cooperative integration of aircraft systems. Without a preliminary control system design, they are unable to approximate the closed-loop dynamic performance—an absolute necessity for the analysis of any high performance aircraft. Future goals seek to incorporate even more advanced control strategies through the implementation of the Variance Constrained Flying Qualities approach. Application of the VCFQ approach will first be performed on the subsonic transport aircraft discussed herein, and then efforts will focus on the integration of the VCFQ-based stability and control module into the ESAV MDO framework.

References


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