DEVELOPMENTS IN WIDEBAND TIMING OF MILLISECOND PULSARS

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ABSTRACT

Pulsar timing is an extremely powerful technique in the field of observational astronomy, particularly when applied to millisecond pulsars (MSPs). In effect, MSPs act as precise laboratory clocks with which one can make precise physical measurements via pulsar timing. However, the recent development of wideband, low-frequency receiver systems that allow simultaneous observation of much of the radio spectrum relevant for timing have produced challenges for conventional timing methods. The development of improved methods should provide opportunities to better study pulsar emission, the magnetosphere, and the interstellar medium (ISM). In this work we present the first stages of a new timing algorithm that makes use of a global two-dimensional model of the pulsar emission to produce timing and ISM measurements. We demonstrate the validity and effectiveness of the new algorithm on a test MSP, the pulsar M28A, and compare our findings with those of standard techniques. By naturally incorporating profile evolution due to the ISM and intrinsic profile changes, the new methodology should remove some systematic error and possibly increase the precision of timing MSPs, which is important for high-precision experiments like those aimed at making the first detections of gravitational waves.

1. INTRODUCTION

Pulsars are rapidly rotating, highly magnetized neutron stars that produce anisotropic, beamed, broadband radiation. If this beam crosses an observer's line-of-sight during rotation, then a periodic recurrence of the “pulsed” radiation can be seen as a function of time. When the time-series is folded modulo the rotational period, the average light curve as a function of rotational phase is called a pulse profile (see Figure 1), which may vary as a function of observing frequency (see Figure 2). The rotational “light-house effect” can be checked against terrestrial clocks as a measure of time. The extreme regularity of the pulses and their broadband nature across the radio spectrum provide means to make a large number of interesting physical measurements, in effect by using the pulsar as a laboratory clock, a process referred to as pulsar timing.

1.1 Pulsar Timing

Pulsar timing has produced a wealth of scientific results, most of which have occurred in the last two or three decades of its 45-year existence. These results include the detection of the first extra solar planet (Wolszczan & Frail 1992), making precise measurements of stellar masses and distances (Deneva et al. 2012; Verbiest et al. 2008), probing the interstellar medium (Walker et al. 2008), placing the best constraints on, and verifying,
General Relativity (Kramer et al. 2006), as well as yielding insights into the interiors and plasma environs of neutron stars (Demorest et al. 2010; Breton et al. 2012), which are some of the most exotic objects in the universe. Specifically, the most famous application of pulsar timing gained Russell Hulse and Joe Taylor the 1993 Nobel Prize in Physics for their indirect detection of gravitational waves (GWs).

The most stably rotating pulsars useful for such experiments belong to a class called millisecond pulsars (MSPs). MSPs are distinct not just because they have the shortest spin periods \( P_s < \sim 20 \text{ ms} \), but because they represent a different phase of pulsar evolution. Normal pulsars have periods near 1 sec, but if a neutron star has a binary companion that undergoes post-main-sequence evolution, it can be “recycled” or “spun-up” to millisecond periods. MSPs are thought to have relatively stable spins and emission for billions of years.

The timing of a pulsar involves making periodic observations of the pulsar and connecting these observations with a model describing how the pulsar spins – the timing model – in order to account for each individual rotation of the neutron star (see Lorimer & Kramer (2008), chapter 8). This “phase-connection” of observations during the time when the pulsar is not being observed justifies the use of the timing model to predict future rotations of the neutron star, since the phase-connected observations imply no rotations have been missed. Timing is a bootstrap process that starts with a minimal number of parameters in the model, such as spin period and position coordinates, and improves with additional observations by adding additional parameters like period derivatives, binary parameters, proper motion, etc. The fundamental timing measurement of interest, from which all other parameters are derived, is the pulse time-of-arrival (TOA), which is an accurate determination of when a pulse has arrived at the observatory, or, more abstractly, the time of passage of an arbitrary line of longitude on the neutron star surface. New parameters are incorporated into the timing model when systematic deviations are observed in the timing residuals, which are the differences between the observed TOAs and the prediction made by the model. The best-timed MSPs will exhibit clock-like behavior for years, using less than two dozen parameters to explain thousands of data points.

### 1.2 Effects of the interstellar medium

The interstellar medium (ISM), which consists of the material between Earth and the pulsar, leaves imprints on the pulsar signal that gets detected at radio frequencies (see Lorimer & Kramer 2005, chapter 4). The primary effect that we are concerned with here is interstellar dispersion, which is quantified by the dispersion measure (DM). The DM of a pulsar is simply the column density of free-electrons \( n_e \) along the line-of-sight,

\[
\text{DM} = \int n_e dl
\]  

(1)
The effect of dispersion on a signal comprised of two radio frequencies, \( \nu_1 \) and \( \nu_2 \), is a relative time-delay that depends on the inverse of the frequency squared,

\[
\Delta t = D_{\text{const}} \times \text{DM} \times (\nu_2^{-2} - \nu_1^{-2})
\]  

(2)
where \( D_{\text{const}} \) is called the dispersion constant. A pulsar's DM can change at a measurable level on at least week or month time-scales (Keith et al. 2013). It is obvious from equation (2) that MSPs will be most sensitive to changes in DM; a small change in DM constitutes a fixed difference in the expected delay between frequencies, but that delay is a larger fraction of the rotational period for fast-spinning MSPs. This means that measurements of the DM must be made for each individual observation of an MSP if one is to obtain accurate timing.

Another important effect of the ISM is scintillation, which is a frequency-dependent, stochastic modulation of the pulse intensity, akin to “twinkling”, that affects nearby pulsars more so than distant ones. Scintillation may
also change from day to day, altering the measured fluxes at particular frequencies, which implies that some frequencies may contain more signal on one day, and less the next. To avoid this problem, simultaneous observation of a range of frequencies over a large bandwidth is desired.

Lastly, multi-path propagation through the ISM results in the broadening of the pulse profile. Simple models of this phenomenon called scattering suggest that it has a strong $\nu^{-4}$ dependency, and can be considered a convolution of the unaltered profile with a one-sided exponential function. Generally speaking, scattering effects only become important for distant pulsars observed at low frequencies.

1.3 Motivation for development

MSPs offer the promise of the first direct detection of gravitational waves by way of a Pulsar Timing Array (PTA) (Hobbs et al. 2010). PTA efforts involve monitoring a number of well-timed MSPs across the galaxy in order to look for specific correlated signals in their residuals that are obtained from long-term timing campaigns at large single-dish radio telescopes. That the pulsars' residuals are correlated implicates a terrestrial perturbation, which could be due to the passage of GWs that alters Earth's local spacetime. The first detection of GWs will represent another critical test of fundamental physics, and an opening of a new window into the universe, as GWs provide information that is not measurable by the detection of electromagnetic radiation. PTA experiments are sensitive to GW frequencies in the nanohertz regime, which corresponds to the years to decades of timing data collected from the MSPs. Laser-interferometric GW detection efforts are sensitive to completely complimentary frequency regimes.

The gravitational wave signal present in PTAs is thought to require timing precision of order hundreds of nanoseconds for detection. However, the precision to which a pulsar can be timed may be artificially limited by marginalizing over frequency-dependent evolution of the profile due to propagation effects through the ISM, or possible intrinsic profile dependency on observing frequency (Ahuja et al. 2009; Hassall 2012; You et al. 2007). Early pulsar observations were confined to narrow bandwidths, typically 10s of MHz, and the basic timing methods applied to these older datasets could either ignore these frequency-dependent effects, or treat them separately from the TOA measurements. With the recent advent of wideband receiver systems that allow for instantaneous observation of ~1-3 GHz of bandwidth of the pulsar's signal, it becomes imprudent to ignore the frequency evolution of the pulse profile when making TOAs. In practice, it is extremely difficult to disentangle certain ISM effects from intrinsic profile evolution (Hassall et al. 2012), which is a problem that can potentially be resolved by the development of a global timing algorithm.

Therefore, in order to obtain ever-greater precision and accuracy in pulsar timing, particularly for the sake of improving sensitivity to the detection of GWs by using a PTA, we seek an improved timing methodology that naturally incorporates changing profiles with frequency, appropriate for use with modern and all anticipated pulsar instruments.

1.4 Outline

In section 2, we present the standard timing methodology, our simple extension to it, and how we implement it. In section 3, we present the application of the new algorithm to a test pulsar, M28A, to demonstrate its validity. In section 4 we briefly discuss future applications and development, and conclude.

2. TIMING METHODOLOGY

Pulsar data are time-series data taken by a backend instrument that records digital signal values fed in from a particular telescope and receiver combination. The receiver is sensitive to a bandwidth of frequencies that gets split up into some number of discrete frequency channels. After applying a correction for an assumed DM value to each channel, a process called coherent dedispersion, these time-series
are coherently “folded” based on the timing model, which is imperfect. The folded pulsar timing data can be most easily visualized as a cube of flux density measurements, where the flux density measurements are made as a function of observing time, observing frequency, and rotational phase. Polarization information is often collected, but we will ignore this fact and consider only total intensity observations that have been properly polarization calibrated. The folded data are then used to generate TOAs, which is the critical step in the process of timing that we seek to improve upon.

2.1 Basic TOA fit procedure

The fit for a TOA involves taking a time- and frequency-average of the data – a profile – and maximizing its cross-correlation with a standard “template” profile to measure a lag, or phase offset. In order to get an absolute time-stamp for the TOA, the phase offset is related back to the integer number of that particular pulse profile and the absolute measured time in some frame of reference, such as Universal Coordinated Time or the solar-system barycenter. Since the publication of Taylor (1992), the cross-correlation has been equivalently, but advantageously implemented in the Fourier frequency domain as a multiplication of the template with a complex exponential in order to avoid binning effects. The algorithm came to be known as FFTFIT.

The quantity to be minimized is

$$
\chi^2(\phi, a) = \sum_k \frac{|d_k - ap_k e^{-2\pi i k \phi}|^2}{\sigma^2},
$$

where $d_k$ is the Discrete Fourier Transform (DFT) of the profile, $p_k$ is the DFT of the template, $k$ indexes the harmonic bin number, $\sigma$ is the noise level in the Fourier-transformed profile (assumed to be independent of phase), $a$ is the fitted amplitude, and $\phi$ is the fitted phase. That is, one assumes the data profile is just a scaled, shifted version of the template profile with added noise. $a$ is essentially a nuisance parameter and can be trivially, analytically calculated from the best-fit value of $\phi$. It should be noted that, although not important here, $a$ represents a measure of the scintillation for that epoch and frequency. Generally speaking, the template profile can be anything, but in practice one uses a high signal-to-noise, smoothed average of all the data available. It is evident from this expression that no frequency information is incorporated into the fit. In practice, one makes TOAs from profiles at different frequencies, and these TOAs do not necessarily have any natural relation to one another. In fact, one generally uses different template profiles for different frequencies since intrinsic pulse profile changes, scintillation from the ISM, or significantly degraded signal-to-noise can alter the pulse profile shape. These unrelated TOAs, which will have unknown, arbitrary phase offsets relative to one another, are then usually used to measure a DM for their shared (or quasi-shared) epoch. An ad hoc method like this, designed for when observing bandwidths were narrow and frequency bands were effectively spaced far apart, introduces unknown systematics into the timing residuals, ultimately degrading the ability to make high-precision measurements with the pulsar clock, and is simply inconvenient.

2.2 Extension to FFTFIT

The natural extension to the Taylor (1992) expression that we present below links profiles of different frequencies – and therefore their shared, fitted TOA – across an arbitrarily large bandwidth via their relation to the simultaneously-fitted DM and the use of a global model template. The quantity that we now seek to minimize in a fit is analogous in form, but carries an additional index $n$, that indexes the discrete frequency channels,

$$
\chi^2(\phi, \text{DM}, a_n) = \sum_{n,k} \frac{|d_{nk} - a_n p_{nk} e^{-2\pi i k \phi_n}|^2}{\sigma_{n}^2},
$$

where $\phi_n$ is the sum of the overall achromatic delay $\phi$, which is related to the TOA as...
described before, and the dispersive phase delay from equation (2),

$$\phi_n = \phi + D(\nu_n^2 - \nu_{ref}^2).$$

Here, $\nu_{ref}$ is the arbitrary reference frequency that is defined to have zero delay from a non-zero DM, and the pre-factor $D$ is the combination of

$$D = \frac{D_{\text{const}} \times \text{DM}}{P},$$

where $P$ is the pulsar's spin period. There are a few important differences to note in our extension of FFTFIT. The data $d_{nk}$ and model $p_{nk}$ are now collections of phase-vs-frequency profiles, which we call portraits, that have been DFT'd along the phase axis. The portraits retain useful information stored in the frequency structure of the pulsar signal for the fit. It is likely that the noise levels are different in each channel across a wide bandwidth, especially in the presence of narrow-band radio frequency interference (RFI), so we have included the channel-indexed noise parameter $\sigma_n$. The $a_n$ are scaling parameters per channel and, similarly to before, can be calculated analytically after fitting for $\phi$ and DM. However, in this case, the $a_n$ automatically give more weight to frequency channels with greater signal based on the random scintillation pattern, as well as the real power-law-like nature of the flux in the profile components, ensuring an appropriate fit for the TOA and DM. That the DMs are measured in situ with the TOAs implies that an immediate handle on the temporal variability of the DM is available after measuring TOAs. In practice, the only timing artifacts that might remain from interstellar dispersion would originate in not coherently dedispersing within a discrete channel at the exactly correct DM for that epoch. Hence, there will always be some unavoidable residual smearing in each frequency channel due to dispersion. As before, the model $p_{nk}$ for the pulsar signal is, in general, arbitrary and can be constructed to include any inferred profile evolution. In principle, the model can straightforwardly include the effect of scattering, which is virtually constant as a function of time. Therefore, the new algorithm has the ability to naturally account for the major ISM effects while producing a single TOA value, which is the most important number for timing purposes.

2.3 PulsePortraiture

We have implemented the algorithm described above in the python\(^2\) programming language by extensive use of the numpy\(^3\) and scipy\(^4\) libraries, as well as the python interface to the pulsar data analysis software package called PSRCHIVE\(^5\). The code, which we have collectively named PulsePortraiture, is publicly available online\(^6\). The routine pptoas.py measures TOAs and DMs given data in PSRCHIVE format, and a template portrait. The additional step we have had to improvise is the creation of the template portrait. Unlike the narrow-band case where one can simply marginalize over frequency and average all available data to make a high signal-to-noise, one-dimensional profile, the averaging of all the data portraits available would necessarily smear out the signal because of the slightly different DMs in each epoch, making the fit for individual DMs less sensitive. Nevertheless, we have incorporated functionality to allow arbitrarily averaged and smoothed portraits to be used as templates.

For the applications described in the next sections, we have decided to proceed by generating templates based on the decomposition of the pulsar portrait into gaussian components that evolve with frequency. Similar methods have previously been employed to model profile evolution (Lommen 2001, Hassall et al. 2012), although MSPs have not been studying in this way in great detail. The routine ppgauss.py accomplishes this by fitting gaussians to a

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2 http://www.python.org/

3 http://www.numpy.org/

4 http://www.scipy.org/

5 http://psrchive.sourceforge.net/

6 https://github.com/pennucci/PulsePortraiture
profile from real data at some reference frequency, and then allowing the components to change independently in height, position, and width. The fitting of the model components is highly flexible, allowing for independent fitting for different parameters for different components, and attempts to converge by fitting out any detected DM in the model.

3. APPLICATION TO M28A

The millisecond pulsar J1824-2452A, also known as M28A (Lyne et al. 1987), is a perfect case to demonstrate the utility of the new algorithm. M28A is a fast MSP, with $P_s = 3.05$ ms. It is bright, having a peak flux density at 1500 MHz of around 30 mJy. It is located in the globular cluster M28, which has a large DM of $\sim 119$ cm$^{-3}$ pc. More importantly, it is known to show relatively large DM gradients with time, of order $3 \times 10^{-3}$ cm$^{-3}$ pc yr$^{-1}$ (Keith et al. 2013). It displays a complex profile shape (Figure 1), whose three main components change in relative amplitude with frequency (Figure 2). However, it is also one of the rare instances of an isolated MSP, which are usually found in binary systems and could complicate the basic timing procedure. Lastly, we have collected a substantial amount of data on M28A using the 100-m Robert C. Byrd Green Bank Telescope (GBT)$^7$ in tandem with the backend instrument called the Green Bank Ultimate Pulsar Processing Instrument (GUPPI)$^8$.

3.1 Timing residuals

We have tested the consistency of the phase measurements between the two methods. Figure 3 shows an example of residuals obtained from fitting the timing model in Table 1 to TOAs measured within a single epoch every five minutes, using each method. The TOAs follow the same systematic jitter, although the error bars estimated from our new

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$^7$ The GBT is operated by a cooperative agreement of the NRAO and AUI, and is funded by the NSF.

$^8$ https://safe.nrao.edu/wiki/bin/view/CICADA/GUPPiUsersGuide
Table 1 – The timing model for M28A. F0 is the spin frequency; F1 is the first spin frequency derivative; PEPOCH is the epoch of F0; RA/DEC are the positional sky coordinates; μ are the proper motions along the sky coordinates, DM is as in the text.

<table>
<thead>
<tr>
<th>PSR</th>
<th>J1824–2452A (M28A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F0</td>
<td>327.405606052434</td>
</tr>
<tr>
<td>F1</td>
<td>-1.7353552×10^{-20} Hz s(^{-1})</td>
</tr>
<tr>
<td>PEPOCH</td>
<td>53800.000000</td>
</tr>
<tr>
<td>RA</td>
<td>18(^{h})24(^{m})32.008341(^{s})</td>
</tr>
<tr>
<td>DEC</td>
<td>-24(^{h})52(^{m})10.913(^{s})</td>
</tr>
<tr>
<td>μ(_{RA})</td>
<td>-0.97 mas yr(^{-1})</td>
</tr>
<tr>
<td>μ(_{DEC})</td>
<td>5.5 mas yr(^{-1})</td>
</tr>
<tr>
<td>DM</td>
<td>119.8964 cm(^{-3}) pc</td>
</tr>
</tbody>
</table>

Figure 3 – The black crosses are residuals from TOAs obtained using the conventional technique, and the red x's are from the new method. Note that the residuals show shared systematic trends, and that the red points have underestimated errors.

Figure 4 – The residuals from the whole data set of M28A observations obtained by the GBT+GUPPI at L-band that have been coherently dedispersed. The weighted RMS residual is 320 ns.

3.2 DM measurements

Similarly important is the test that our DM measurements are consistent with what others have found. Figure 5 shows the agreement, with the same key as Figure 3. However, in this case as well, our error bars are underestimated. It should be noted that our measured DM trend (the shape, scale and temporal position) is entirely consistent with that measured for M28A by a separate group using the 64-m Parkes radio telescope in Australia (Keith et al. 2013), which is an independent verification of our measurements.

4. Future Development

Despite having demonstrated that our new timing algorithm is capable of producing reliable TOAs and measurements of DM consistent with other methods, we have yet to show unequivocally in which regimes the new

fitting scheme are likely underestimated by a factor of 2-3.

Figure 4 shows the timing residuals from all of the coherently dedispersed L-band data from the GBT+GUPPI. Each epoch is approximately a 2.5 hr observation, which has

been divided into ~30 5-minute integrations for which a TOA and DM has been measured as described. However, when the timing model was fitted to obtain the residuals, a single per-epoch DM was used for the group of TOAs by averaging the individually measured DMs per TOA within each epoch. The DM in Table 1 is the nominal DM for M28A against which all the other DMs were measured, and was not fit in the timing model. The typical error bars are of order 150 ns, which are underestimated by a factor of 2-3; the weighted RMS residual is approximately 320 ns. If the epoch-dependent DM is not corrected for, and the same timing model is applied with the single listed DM value, the residuals show random systematic trends and have a weighted RMS of ~1.2 μs, a factor of 4 worse. The systematics present in the residuals of Figure 4 are likely a combination of incorrectly determined proper motions and/or the known timing noise that exists in M28A (Rutledge et al. 2003). All of these results are consistent with those found in a separate analysis by utilizing only conventional methods.


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methodology is advantageous. It is certainly the case that the method presented here is conceptually cleaner in making TOAs and measuring DMs, but it is also necessarily more complicated in determining a template. Furthermore, because of the much larger number of data points to be considered in the fit (which multiplies by the number of frequency channels), and because of the additional non-trivial numerical computation involved in the fit, the speed at which TOAs are made is considerably slower, although not prohibitively so.

Ongoing development of the python implementation of the algorithm presented here will include investigating more robust model template generation schemes, the explicit incorporation of scattering into the model, speeding up the TOA fitting routine, making the fits distributable over a cluster for fast, parallel processing, and exploring possible ways to use polarization information for better timing.

The next immediate application of the algorithm will be to characterize the DM variations and to augment, and perhaps improve, the timing of MSPs in the PTA effort represented by the NANOGrav collaboration, in which the authors take part. A manuscript of the research described here for submission to a peer-reviewed journal is currently being written.

It is worth noting that the maturation of wideband timing, as a general practice, has the potential to lead not only to the best timing from MSPs to date, but can also lead to better characterization of the turbulent ISM by the increasingly regular measurements made of DM and scattering effects, as they will necessarily become part of TOA production. Ambitiously, one might also hope to gain insight into the structure and workings of the pulsar magnetosphere by removing ISM effects in order to study the frequency evolution of broadband pulsar emission.

4.1 Acknowledgements

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