MATHEMATICAL MODELS OF FLUTTERING PLATES: SUPERSONIC FLOWS

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Abstract

The *flutter phenomenon* is the principal concern in the field of aeroelasticity. In this paper we address the fundamental questions of well-posedness and long-time behavior of the primary mathematical model for a *fluttering plate* in a panel configuration. Such a system is known as a *nonlinear flow-plate interaction*. The flow is described by a perturbed wave equation. The plate is modeled by Kirchoff's plate equation; we consider various configurations for the plate and focus on the primary *physical* configuration from large deflection theory utilizing the *von Karman nonlinearity*. Strong coupling between the two dynamics occurs in the acceleration potential for the plate and the down-wash for the flow (Neumann type flow condition).

We provide a discussion of the most recent mathematical results pertaining to these issues in the *principal case* of interest (both mathematically and in application): supersonic flow velocities. Specifically, we provide a discussion and interpretation of results on semigroup well-posedness and existence of global attractors for flow-plate models, as well as a discussion of future directions for mathematical analysis for this broad class of models.

Key Terms: aeroelasticity, flutter, fluid-structure interaction, nonlinear PDE, control theory, long-time behavior, dynamical systems

1 Introduction

1.1 Background

1.1.1 Physical Motivation

In this treatment we continue a discussion of recent studies of the mathematical model for a fluttering plate, as introduced in bolotin,cole,dowell, [4, 13, 14, 15, 16] and addressed by the author in the proceedings of the 2012 VSGC Research Conference vsgc [27]. The *flutter pheonmenon* is of great interest, in particular, in the field of aerospace and aeroscience. Flutter is a sustained endemic instability which occurs as a potentially violent feedback between a structure and an inviscid fluid when the natural modes of a thin structure "couple" with an fluid's dynamic load. When a structure is immersed in an inviscid fluid flow, certain flow velocities represent a bifurcation in the dynamics of the coupled *flow-plate* system wherein stable, non-oscillatory dynamics may become oscillatory (limit cycle oscillations) or chaotic. The phenomenon occurs in a multitude of applications, including (but not limited to): buildings and bridges in strong winds, panel and flap structures on air and land vehicles, and in the human respiratory and circulatory systems. The famous Tacoma Narrows Bridge fell victim to the flutter phenomenon in 1940 resulting in complete structural failure and the ultimate collapse of the bridge. Recently, flutter resulting from *axial flow* (which can be achieved for low flow velocities) has been studied from the point of view of energy-harvesting (via piezoelectric materials) energyharvest [17].

From a design point of view, flutter cannot be overlooked due to its potentially disastrous effects on structure due to sustained fatigue or structural failure. The experimental and theoretical questions revolve around the prediction and suppression of flutter for a given physical system. The field of *aeroelasticity* is concerned with (1) producing models which describe the flutter phenomenon, (2) gaining insight into the mechanism of *flutter*, (3) predicting the behavior of a flow-structure system based on its physical parameters and configuration, and (4) determining appropriate control mechanisms and their effect in the prevention or suppression of instability in the flow-plate system.

1.1.2 Mathematical Motivation

Mathematically, a standard model for a fluttering plate or shell has been utilized in an immense body of work produced in the past 60 years. The model was introduced by [4] in the early 50's and has since been expanded and refined; however, most work on aeroelastic models has been *numerical* in nature [13, 14, 15, 16]. And most investigations of flutter and flow-structure interactions have involved an interplay between experimental and numerical analysis (both linear and nonlinear) [14, 15]. However, owing to immense experimental cost and requisite capital, and the potential problems associated to "approximate" numerical models (discussed below), there is strong motivation for analyzing general mathematical flutter models. In this way we can pursue many avenues of inquiry by strictly analyzing the (generally accepted) mathematical model at little or no experimental cost. Additionally, we may (1) make predictions about the behavior of certain physical configurations for flow-structure systems, (2) corroborate experimental and numerical findings, and (3) pursue the mechanism of flutter directly from the model, as derived from first principles.

As previously stated, much of the body of work dedicated to aeroelastic models is numerical in nature. We refer to [13, 14, 15] for an eloquent description of such findings. While numerical methods address many aspects of the problem, they are based on *finite-dimensional approximations* of a continuum model fully described by partial differential equations (PDEs) [4, 5, 2]. The PDE nature of the physical phenomena may not be adequately reflected by these approximations; this is particularly true when dealing with oscillatory models, where large frequencies, often causing instability, cannot be accounted for.

Theoretical results derived from PDE analysis are valuable for a variety of reasons: (1) In feasibility studies (prototypes), one wants to know the regime in which flutter occurs. This knowledge can guide and streamline experimental and numerical flutter threshold determination (2) Such knowledge can improve cost-effectiveness of experimentation, hone-in numerical analysis, and cut-down on design time. (3) Investigating controls for the model can indicate what types and locations of damping will be effective for a given configuration.

1.1.3 Previous Work

In the work funded by the Virginia Space Grant Consortium during the years of 2011–2012 and 2012–2013, we have been interested in providing quantitative analysis of a model which arises in aeroelasticity and is governed by suitable PDEs describing the interactive dynamics between an oscillating structure and a surrounding inviscid flow. We study PDE solutions describing a flexible immersed in an inviscid flow of a fixed velocity (subsonic or supersonic). Specifically, we are interested in the well-posedness, control, and long-time behavior of a coupled partial differential equations model which we introduce in Section 1.2 below.

In the the previous academic year, 2011-2012, the PI pursued the question of well-posedness and long-time behavior of the dynamical system corresponding to the flow-plate model *in the case of subsonic flows* for a panel configuration, and in a configuration involving a nonlinear dissipation mechanism present on the boundary of the plate. Specific statements of these results can be found in the manuscripts [26, 19, 20]. For a general overview of the mathematical results for the model presented below (in many configurations), see the VSGC research manuscript by the author for the 2011-2012 academic year [27], and the references therein.

In the present consideration, we focus on the most difficult configuration (case): supersonic flow velocities. From the point of view of aeroelasticity and application, this is also the most pertinent case. At the time of the previous manuscript in the 2011-2012 year, the case of supersonic flow velocities was essentially open. Almost no work had been performed owing to the fact that the model of interest had yet to be shown to be *well-posed* (although this is addressed in [27]). We address well-posedness in more detail below, however, we suffice to say here that if a mathematical model is well-posed, this means it represents a physical model which is, to a certain degree, viable for making predictions about the physical system. Without well-posedness of a mathematical model, no control theoretic or long-time behavior studies can be justifiably made from the point of view of partial differential equations and dynamical systems modeling. Moreover, well-posedness results are important for guiding and providing justification for numerical studies of a given model. Having obtained well-posedness for the supersonic flow-plate model in [10] near the end of the 2011–2012 academic year, many compelling mathematical questions arose which we have been able to address and provide herein a relevant summary and discussion.

1.2 Mathematical Model

The model in consideration involves the interaction of a plate (with standard boundary conditions [22]) with a field or flow of gas above it (we need only consider the flow on one side of the plate by antisymmetry). To describe the behavior of the gas, we make use of the theory of potential flows [4, 14, 15]. The oscillatory dynamics of the plate are governed by second-order in time nonlinear plate equations. For the plate, we consider the most physically relevant nonlinearity: the von Karman nonlinearity [12] of recent interest [8] (and the many references therein); this model plate dynamics with 'large' displacements, and therefore appropriate for flexible structures of interest. Including nonlinearity in the structure of the model will turn out to be critical, not only for the sake of accuracy in modeling, but also because nonlinear effects play a major role in "bounding" the energies associated to the coupled dynamics [8].

The environment we consider is $\mathbb{R}^3_+ = \{(x,y,z) \in \mathbb{R}^3 \big| z \geq 0\}$. The thin plate has negligible thickness in the z-direction (as is usual in the modeling of thin structures [12]). The unperturbed flow field moves in the negative x-direction at the fixed velocity U, with $U = 1$ corresponding to the speed of sound. The top surface of the plate will be denoted $\Omega \subset \mathbb{R}^2$ and is assumed to be bounded in $\mathbb{R}^2$ with smooth boundary. The scalar function $\phi(x,y,z;t)$ gives the flow potential. The scalar function $u(x,y;t)$ represents the z displacement of the plate.

Our coupled system is as follows (taking $\mathbf{x} = (x,y,z)$ or (x,y), as dictated by context) and ν to be the outward normal direction to $\partial\Omega$: First, the nonlinear Kirchoff plate equation with clamped boundary conditions is given by

$$\begin{cases} u_{tt} + \Delta^2 u + f(u) = p(\mathbf{x},t) \text{ in } \Omega \\ u = \partial_\nu u = 0 \text{ on } \partial\Omega \\ u(t=0) = u_0(\mathbf{x}),\ u_t(t=0) = u_1(\mathbf{x}). \end{cases} \tag{1}$$

Here $u_0,\ u_1$ are initial data. Other boundary conditions we are interested in include: clamped, hinged, and clamped-free and clamped-hinged. These will be explicitly stated later in the treatment when pertinent results are discussed. The quantity $p(\mathbf{x},t)$ corresponds to the aerodynamical pressure of the flow on the plate and is given in terms of the flow $p(\mathbf{x},t) = p_0 + (\partial_t + U\partial_x)\phi|_{z=0}$, $\mathbf{x} \in \Omega$, which is known as the *acceleration potential* of the plate. The quantity p_0 represents static pressure applied on the surface of the plate.

Remark 1.1 *A parameter $\gamma \geq 0$ may be incorporated into the model by replacing the term u_{tt} with $(1-\gamma\Delta)u_{tt}$; this term γ represents rotational inertia in the filaments of the plate and is proportional to the cube of the thickness of the plate. The case of in-*

terest, $\gamma = 0$*, is said to be* irrotational *and is the most interesting mathematically and physically (see [13] for discussion).*

The von Karman nonlinearity ($f(u) = f_V(u)$) is defined it in terms of Airy's stress function $v(\cdot)$ [8]: $f_V(u) \equiv -[v(u) + F_0, u]$, with $[\cdot,\cdot]$ denoting the von Karman bracket [8], and F_0 a given in-plane load.

Secondly, a perturbed wave equation describes the flow:

$$\begin{cases} (\partial_t + U\partial_x)\phi = \Delta\phi \text{ in } \mathbb{R}^3_+ \\ \partial_z\phi = \begin{cases} (\partial_t + U\partial_x)u \text{ in } \Omega \\ 0 \text{ off } \Omega \end{cases} \end{cases} \tag{2}$$

The coupling between the plate and the flow occurs in the Neumann boundary condition, in the 'down-wash' of the flow.

Remark 1.2 *The key parameter in our analysis is* U*, with regimes* $0 \le U < 1$ *(subsonic) or* $U > 1$ *(supersonic)—additionally, the value* $U = 1$ *is referred to as the* transonic barrier*. This parameter greatly affects the dynamics of the model, and many results discussed in [27] and below (and their proofs) depend critically on the regime in which we are working. The principal regime - the most interesting from the applied point of view, and most challenging mathematically - is* $U > 1$*.*

Remark 1.3 (Notation) *above and for the remainder of the text, norms* $||\cdot||$ *are taken to be* $L_2(D)$ *for the domain dictated by context. Inner products in* $L_2(\mathbb{R}^3_+)$ *are written* $(\cdot,\cdot)$*, while inner products in* $L_2(\Omega)$ *are written* $<\cdot,\cdot>$*. Also,* $H^s(D)$ *will denote the Sobolev space of order* s*, defined on a domain* D*, and* $H^s_0(D)$ *denotes the closure of* $C^\infty_0(D)$ *in the* $H^2(D)$ *norm*

Finite energy constraints (as dictated by the physics of the model) manifest themselves in the natural requirements on the functions ϕ and u: $u \in C(0,T; H^2_0(\Omega)) \cap C^1(0,T; L^2(\Omega))$; $\phi \in C(0,T; H^1(\mathbb{R}^3_+)) \cap C^1(0,T; L^2(\mathbb{R}^3_+))$. Moreover, to set up the model in a dynamical systems framework, we will take our state space to be $Y = Y_{fl} \times Y_{pl} \equiv \big(H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+)\big) \times \big(H^2_0(\Omega) \times L_2(\Omega)\big)$ for the state variable $y = (\phi, \psi; u, v)$, where $\psi \equiv (\phi_t + U\phi_x)$.

We note that considering supersonic flows (taking $U > 1$ in the flow equations) corresponds to the loss of strong ellipticity in the spatial flow operator. This leads to a degenerate static problem in semigroup considerations for the flow, which is the predominant issue in the well-posedness analysis of the flow-plate system.

1.3 Basic Notions of Interest

1.3.1 Well-posedness

For a given PDE model, the primary consideration is that of (Hadamard) well-posedness. For a PDE or PDE system, well-posedness refers to the *existence, uniqueness, and continuous dependence on initial data* of finite energy solutions to the system for an arbitrary interval of time $[0,T]$. In this case, we refer to the semigroup well-posedness of the system, namely the existence of a strongly continuous C_0 semigroup on the state space Y as described above. These notions of well-posedness and the existence of a C_0 semigroup can be thought to coincide in most cases of import. It is necessary when discussing well-posedness to a PDE or system of PDEs to specify what *type* of solution is being sought. In our case, finite energy solutions are identified with so called mild (or semigroup) solutions, which can again (often) be identified with *weak* solutions. In our exposition below, we are careful to specify which type of solutions pertain to a given result; however, for an explicit description of types of solutions for the model above in (1)-(2), please see [26, 8, 20, 10, 9].

These somewhat identical notions of well-posedness and the existence of a semigroup are important in applying dynamical systems considerations to the analysis of the model. Many techniques and powerful theorems are available for PDE systems which generate dynamical systems (evolutions, semigroups, and semiflows). In general, a **dynamical system** is a pair (X, S_t)

where X is a separable metric space and S_t is the so-called evolution operator; for each $t \in \mathbb{R}_+$, S_t is a continuous mapping from X to X, whose defining properties are the semi-group properties: • $S_0 = Identity$, • $S_{t+s} = S_s \circ S_t$.

1.3.2 Long-time Behavior

Having defined a dynamical system, we will now discuss and define a few basic notions in dynamical systems theory which will be needed to state our results later in the text. The principal study of long-time behavior involves the stability of a system, specifically, the convergence of trajectories of the dynamical system to points of equilibria. This investigation is nontrivial, especially in the case of nonlinear dynamical systems.

First, a **dissipative** dynamical system (X, S_t) is a dynamical system with a uniform absorbing set D such that for any bounded set $B \subset X$ we have that $S_t B \subset D$ for t sufficiently large. The dynamical system is called **compact** if the absorbing set is compact in the state space X. (X, S_t) is said to be **asymptotically compact** if there exists a compact set $K \subset X$ which is uniformly attracting (see [8] for discussion of the terminology). A dynamical system is said to be **asymptotically smooth** if for any bounded, forward invariant set D, there exists a compact set $K \subset \overline{D}$ which is uniformly attracting. Lastly, a **global attractor** is an invariant set for the dynamics which is uniformly attracting.

After showing the existence of an attracting set for the plate dynamics (if possible), often the next step is to show that the ultimate attracting set for the dynamics has "nice" properties—namely, finite-dimensionality and additional regularity. Ultimately, this is tantamount to reducing the infinite dimensional dynamics of a PDE system to a finite dimensional set of additional regularity (above that of the state space) to which finite dimensional control theory can be applied.

2 Well-posedness of the Mathematical Model

We do not provide a complete discussion of previous well-posedness analysis, but rather refer the reader to [7, 10] for a rather complete description of the current results. We suffice it to say that, to date, the issue of well-posedness of von Karman (and more general, physical, nonlinear flow-plate interactions) was completely resolved for all parameter combinations of U and γ, except in the case $\gamma = 0$ and $U > 1$. In fact, this *principal* case was mentioned in [7, 26, 8, 27] as the primary case of interest, to which no work had been done. In the following section, we discuss the novel result recently obtained in [10] which resolves the question of well-posedness in the case of supersonic flows, and completes the analysis of well-posedness for the flow-plate systems.

2.1 Technical Description of Results

Recall, our state space for the analysis to follow is

$$Y \equiv H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times H^2_0(\Omega) \times L_2(\Omega).$$

Theorem 2.1 (Linear) *Consider linear problem in (1)-(2) with $f(u) = 0$. Let $T > 0$. Then, for every initial datum $(\phi_0, \phi_1; u_0, u_1) \in Y$ there exists unique generalized solution*

$$(\phi(t), \phi_t(t); u(t), u_t(t)) \in C([0,T], Y) \quad (3)$$

which defines a C_0-semigroup $T_t : Y \to Y$ associated with (1)-(2) (where $f = 0$).

For any initial data in $Y_1 \equiv$

$$\left\{ y \in Y \;\middle|\; \begin{array}{l} \phi_1 \in H^1(\mathbb{R}^3_+), \quad u_1 \in H^2_0(\Omega), \\ -U^2\partial_x^2\phi + \Delta\phi \in L_2(\mathbb{R}^3_+), \\ \partial_z\phi = [u_1 + U\partial_x u]\mathbf{1}_\Omega \in H^1(\mathbb{R}^2), \\ \quad -\Delta^2 u + U\gamma[\partial_x\phi] \in L_2(\Omega) \end{array} \right\} \quad (4)$$

the corresponding solution is also strong.

Theorem 2.2 (Nonlinear) *Let $T > 0$ and let $f(u) = f_V(u)$ be the von Karman nonlinearity. Then, for every initial data $(\phi_0, \phi_1; u_0, u_1) \in Y$ there exists unique* *generalized solution $(\phi, \phi_t; u, u_t)$ to (1)-(2) possessing property $(\phi(t), \phi_t(t); u(t), u_t(t)) \in C([0,T], Y)$. This solution is also weak and generates a nonlinear continuous semigroup $S_t : Y \to Y$ associated with (1)-(2).*

If $(\phi_0, \phi_1; u_0, u_1) \in Y_1$, where $Y_1 \subset Y$ is given by (4), then the corresponding solution is also strong.

Remark 2.1 *In comparing the results obtained with a subsonic case, there are two major differences at the qualitative level: (1) Regularity of strong solutions obtained in the subsonic case [26] coincides with regularity expected for classical solutions. In the supersonic case, there is a loss of differentiability in the flow in the tangential x direction.*

(2) In the subsonic case one shows that the resulting semigroup is bounded in time, see [8, Proposition 6.5.7] and also [9, 26]. This property could not be shown in this analysis, and most likely does not hold. The leak of the energy in energy relation can not be compensated for by the nonlinear terms (unlike the subsonic case).

2.2 Qualitative Discussion of Results

From a mathematical point of view (Hadamard) well-posedness for an evolution system means that finite-energy (physical) solutions exist and are unique for some interval of time. Hence the mathematical model has viability and physical correspondence with the phenomena associated to flow-structure interaction - including flutter. Furthermore, a PDE model does not have the deficiency of studying only a finite number of aeroelastic modes, like numerical approximations. Having a well-posed model at the PDE level indicates that the problem is robust with respect to finite element models and indicates that approximations (taken appropriately) converge to a unique solution.

With well-posedness of the supersonic model in hand, we seek to determine the flutter threshold for the model—to ascertain aeroelastic modes. We may then investigate what controls can prevent flutter in a desired range of flow velocities. A whole new course of investigation is now possible, as the full PDE model's viability in the supersonic range is now established.

3 Long-time Behavior of Solutions

In the discussion of long-time behavior of trajectories for the flow-plate interactions, very little has been done in the last 20 years. We are aware only of results for $\gamma > 0$ models (for all flow velocities) [5, 6, 7] *or* models which utilize *thermoelastic smoothing* with $\gamma \geq 0$ [24, 25]. In these cases, the existence of compact attractors for the plate dynamics was shown. Furthermore, in the case of $\gamma > 0$ with subsonic flows, [8] has demonstrated convergence of trajectories to *stationary states*, indicating that in this case the end behavior of the trajectories is static. We discuss this in more detail below, but we now present our novel results for the case of $\gamma = 0$ in the presence of any flow velocity (other than $U = 1$).

3.1 Technical Description of Results

Theorem 3.1 *Suppose $0 \leq U \neq 1$, $F_0 \in H^{3+\delta}(\Omega)$ and $p_0 \in L_2(\Omega)$. Then there exists a compact set $\mathscr{U} \subset H_0^2(\Omega) \times L_2(\Omega) = \mathcal{H}$ of finite fractal dimension such that $\lim_{t\to\infty} d_{\mathcal{H}}\big((u(t), u_t(t)), \mathscr{U}\big) = \lim_{t\to\infty} \inf_{(\nu_0,\nu_1)\in\mathscr{U}} \big(||u(t)-\nu_0||_2^2 + ||u_t(t)-\nu_1||^2\big) = 0$ for any weak solution $(u, u_t; \phi, \phi_t)$ to (1)-(2) with initial data $(u_0, u_1; \phi_0, \phi_1) \in Y$,*

$$Y \equiv H_0^2(\Omega) \times L_2(\Omega) \times H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+),$$

which are localized in $\mathbb{R}^3_+$ (i.e., $\phi_0(\boldsymbol{x}) = \phi_1(\boldsymbol{x}) = 0$ for $|\boldsymbol{x}| > R$ for some $R > 0$). Additionally, we have the additional regularity $\mathscr{U} \subset \big(H^4(\Omega) \cap H_0^2(\Omega)\big) \times H^2(\Omega)$.

3.2 Qualitative Discussion of Results

By showing that the PDE model (infinite dimensional) converges to a finite dimensional (compact) *attractor*, this effectively allows us to reduce the analysis of the infinite dimensional, hyperbolic-like, unstable model (asymptotically in time) to a finite dimensional set upon which classical finite dimensional control theory can be applied (and the dimension upper bounded). The need for this analysis to be asymptotic in time is to allow the dynamics to move through the transient behavior (due to the initial configuration); by studying the asymptotics, we are searching for the essence of the (periodic/chaoic) behavior of the evolution (as expected experimentally and numerically). Moreover, this finite dimensional set may correspond to the set of instabilities, which can directly provide information about the *flutter points* of an aeroelastic model.

The first point of interest in the result above on the existence of a compact, finite dimensional, regularized attractor for energy level solutions is that this attractor exists *in the absence of damping imposted on the system*. In other words, without account for mechanical or frictional damping (inherent or imposed) the hyperbolic-like, unstable dynamics converge to the attracting set. This indicates that the long-time behavior of the dynamical system is truly finite dimensional. Secondly, it shows that the flow itself has a stabilizing effect on the plate, as the reduced potential [5, 6, 8] from the flow equation is what provides the mathematical stabilizing mechanism for the plate. This is in stark contrast to models with $\gamma > 0$, where a very strong form of damping must be implemented in order to see convergence to attractors [8].

Another distinction we make here is that in the case of supersonic flows (the result presented above) we cannot (at present) show convergence to equilibria (static states). We conjecture that, in fact, this will not be possible. Instead, we have shown convergence to a compact, regularized attracting set for the plate dynamics—which can include periodic or chaotic trajectories. In contrast, in the subsonic case (with minimal damping) [8] and preliminary studies by the PI indicate that (under reasonable assumptions) individual trajectories converge to *stationary states*. Physically, this means that the end behavior of the flow-plate evolution is static—what is known as *buckling* in large deflection theory. This corroborates experimental findings [14, 15, 16] that flutter does not occur in panels at subsonic speeds.

Additionally, we point out that the result above is only for the dynamics of the plate, and depends on the flow data being localized. This indicates that our end behavior is not uniform with respect to all flow data. However, the assumption that the flow data is negligible outside of a certain radius is certainly a reasonable physical assumption.

4 Future Directions

Convergence to equilibrium for $\gamma = 0$: In the case of subsonic flows, the dynamical system generated by the flow-plate system are *gradient*. This allows us to utilize the "nice" energy relation *in addition to* the reduction result utilized in [11] and [8]. However, in this case, rather than showing only convergence to a compact attractor (which is already known at this point [11], preliminary work indicates that trajectories converge to equilibrium (i.e. stationary states). This corroborates physical findings—namely, that complex, oscillatory or chaotic behavior (flutter) does not occur in the presence of subsonic flows.

Kutta-Joukowski conditions: There are many alternative configurations of the flow-plate interaction which are of interest in aeroelasticity (and beyond). The next step, although mathematically much more involved, is very pertinent in application; the so-called Kutta-Joukowski condition is given by the modified flow boundary conditions:

$$\begin{cases} (\partial_t + U\partial_x)^2\phi = \Delta\phi & \text{in } \mathbb{R}^3_+ \times (0,T), \\ \phi(0) = \phi_0; \quad \phi_t(0) = \phi_1, & \\ \partial_\nu\phi = -(\partial_t + U\partial_x)u & \text{on } \Omega \times (0,T). \\ (\partial_t + U\partial_x)\phi = 0 & \text{on } \mathbb{R}^2\backslash\Omega \times (0,T) \end{cases} \tag{5}$$

These boundary conditions apply near a free edge when the configuration involves a free-clamped plate. Such a configuration is of interest in both the case of normal flows and axial flows [1, 2, 3].

Our recent work demonstrates that the approach taken in the case of a clamped panel in the presence of supersonic flows is amenable to the *subsonic case* for the free-clamped plate, taken with Kutta-Joukowski flow conditions (owing to the similarity of the respective energy balance equations).

Hypersonic considerations: Having established well-posedness for $U > 1$, a natural question concerns the so-called "hypersonic" limit, i.e. $U \to +\infty$. This was studied in the case of $\gamma > 0$ for short time intervals via a change of variable [7] (and references therein), that is, the limiting behavior of solutions on *short* time intervals. However, in general (and for $\gamma = 0$) this consideration is open. The relationship of solutions as $U \to \infty$ with the so-called "piston" theory for flow-plate interactions (based on the so-called *plane sections law* as discussed in [4]) is also a matter of interest. (The plane sections law leads to an aerodynamic load of the form $p^*(\mathbf{x}, t) = p_0(\mathbf{x}) - (\partial_t + U\partial_x)u$.) Namely, we are interested in comparing full flow-plate solutions in the hypersonic limit to solutions of the retarded von Karman plate as the contribution of the retarded potential becomes negligible. As pointed out in [8, p.334], good agreement of solutions to the flow-plate interaction as $U >> 1$, and use of the load based on piston theory, would provide an argument in favor of the validity of the plate sections law in the study of nonlinear dynamics for aeroelastic structures.

Transonic flows: In experiment it is noted that the model(s) discussed above are not accurate; indeed, the flow equations require additional specificity when the flow velocity is near $U = 1$ (the transonic regime). We note that the base model (as presented above) is already interesting, as we see a *degenerate wave equation* when $U = 1$ which will produce diminished regularity in the x direction. Beyond this, the references [14] indicate that the appropriate flow equation necessarily replaces the term $\Delta\phi$ on the RHS of the flow equation with $\Delta\phi - U\phi_x\phi_{xx}$ in the transonic regime. This introduces new problems mathematically which must be addressed. Experimentally, we note that near $U = 1$ there are many peculiarities, including the possibility of hysteresis and various types of instability.

Localized interior damping: (1) In previous considerations [27, 20], we have seen that nonlinear hinged boundary dissipation is adequate for showing local attracting sets for the subsonic flow-structure interaction. However, in order to show global attracting sets for the plate component of the system, we will need to show ultimate dissipativity (as the full system has a non-gradient structure). A next step in this direction would be to analyze the full flow-structure system (in fact for $\gamma \geq 0$) in the presence of localized interior damping near the boundary, as in [18].

Full von Karman equations: In our considerations (as well as most of those previously) we have utilized the 2-D von Karman equations to model the large deflections of the plate in the flow-plate interaction. Two possible directions of generalization are: (1) modeling with more general "shell" geometries, which necessitates the use of Riemannian geometry in the analysis. Secondly, one can consider the so-called *full von Karman* equations which account for in-plane displacement, as well as transverse displacement. The in-plane displacement acts a system of elasticity, and by considering the full equations, the structure of the nonlinearity is in fact better behaved. However, considering this system greatly complicates all aspects of PDE and numerical analysis. Yet, much work has been done in the past 15 years on the full von Karman plate system, so this provides encouragement for applying similar approaches to the full flow-plate system.

5 Conclusions

Modern interest in flow-plate models has been abundant, owing to the range of applications of substantial value to the broader aerospace community (and beyond). A variety of approaches have been utilized, with the principal approach to aeroelastic problems being numerical (computationally tractable, finite-dimensional approximate models) and experimental (wind-tunnel type tests). Given the difficulty of modeling coupled PDEs at an interface, theoretical results have been sparse. Also, the level of generality included in the model has only reached it current state recently. The results by the PI (and coauthors) above demonstrate that the model can be studied from an infinite dimensional point, control theoretic point of view, and moreover that meaningful predictions can be made about the physical mechanism of flow-plate interactions (and flutter—or its absence) *strictly from the PDE model*. Furthermore, full PDE models take into account all modes of oscillation, including those of higher frequencies that can trigger instability; that is, full PDE modeling does not have the deficiency of studying only a finite number of "modes" in the analysis of what is a fully infinite-dimensional model, derived from first principles.

Specific to flight, understanding the flow-structure interaction (stability and long-time behavior) and associated control strategies can make air travel safer, more cost-effective by improving range, and lower aircraft operating costs. These considerations are paramount in the supersonic regime, especially with the renewed interest in transonic flight, flutter analyses must be also be conducted supersonically. Many organizations have recently renewed interests in working on technologies that enable the return of safe, economically viable, and environmentally friendly supersonic jets. Current work has been experimental (wind-tunnel testing). The goal is to predict what conditions and airspeeds induce flutter and utilize control law designers who can create a control to postpone (or suppress) the flutter point.

NASA's Aeronautical Directorate states that NASA is helping create safer, greener and more effective travel for everyone. Our green aviation goals are to enable fuel-efficient flight planning, and reduce aircraft fuel consumption, emissions and noise. We are confident that theoretical studies (and strategies such as ours) based on (1) PDE modeling, (2) control theory, accounting for all vibrational modes of the plate, (3) and approximations applied at the final stage to the controlled model will provide an implementable solution which is accurate enough to account for phenomena not detectable by finite-dimensional, approximate models. Our solutions are cost-effective, and can be tested without expensive instrumentation.

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