SMALL SATELLITE ATTITUDE CONTROL USING LINEAR MOVING
MASS ACTUATORS

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Abstract

The subject of this research is to study the use of internal moving masses as the primary means of attitude control for a small satellite platform. The goal of this research is to validate the dynamic model formulated in [1] which describes the under-actuated rotational dynamics of a 3U cubesat with two internal orthogonal linearly translating masses. Models have been developed to simulate angular velocity profiles based on different moving mass profiles. These numerical simulations have been compared to experiment data using the Whorl-I spacecraft simulator at the Space Systems Simulation Laboratory (SSSL) at Virginia Tech. This simulator provides a near frictionless environment which allows for testing of rotational dynamics in a simulated microgravity environment. In this paper, the trends in the angular velocity profiles obtained from experimental data are compared to results from the numerical simulations and show agreeable results.

Introduction

Moving masses have applications in the control of numerous aerospace vehicles such as ballistic missiles, satellites, reentry vehicles, and autonomous underwater vehicles (AUVs). Internal moving masses are capable of changing the center of mass of a rigid or flexible body. Through movement of these masses, the inertia properties of the body may also be changed. Changing the center of mass can be advantageous for a body moving through a fluid of non-negligible density such as air or water. The center of mass may be precisely shifted in order to alter the aero/hydro-dynamic moment exerted on the body. This allows for accurate steering without the use of control surfaces or thrusting devices. Changing the inertia properties of a body also may be advantageous due to the presence of the inertia terms in the equations of rigid body rotational motion. A simple example is that of the figure skater. When performing a spinning maneuver, the figure skater extends their arms to decrease their rotational speed and retracts their arms to increase it. The same concept may be demonstrated using internal moving masses to regulate the angular velocity of a rigid body.

The use of moving masses as a primary means of control provide several distinct advantages over traditional control methods. Steering of reentry vehicles using moving masses eliminates the need for control surfaces which experience extremely large aerodynamic forces during atmospheric entry. The ability to accurately steer a reentry vehicle also allows the vehicle to precisely hit a given landing ellipse. The inflatable reentry vehicle experiment (IRVE) is currently experimenting with moving a single internal mass to control the aerodynamic moment on the vehicle. Moving masses also have several advantages over traditional methods for controlling spacecraft. Opposed to momentum exchange devices, thrusters, and magnetic torque rods which require continuous operation to maintain a desired attitude, internal moving masses operate in a discrete manner and eliminate the need for hazardous fuel on-
board the spacecraft. This form of attitude control is also not as susceptible to the magnetic field of the Earth as many others are. This allows mission lifetimes to be extended and decreases energy requirements. Moving masses have an increasingly promising future in the control of many aerospace vehicles.

Microgravity at Virginia Tech is an undergraduate student design team that I participate on and we are the first group to obtain experimental data, to the authors‘ knowledge, involving moving masses as a primary means of satellite attitude control. A 18” x 18” x 18” cube was used to model a satellite and three linear actuators were mounted on the inside in order to translate the masses. The goal of the experiment was to prove experimentally that these masses have the ability of exerting net control torques on a rigid body in an environment free of external torques. This experiment was conducted in a microgravity environment aboard a NASA aircraft during the summer of 2012. Due to drift on the aircraft, some experimental data was contaminated and we determined that a more controlled environment was necessary to conduct the experiment and obtain reliable data. Because the rotational dynamics are of main interest to spacecraft attitude control, the Whorl-I spacecraft simulator in the Virginia Tech Space Systems Simulations Laboratory provides an excellent testbed. Because the simulator has only three degrees of freedom (roll, pitch, and yaw), translational motion is not possible and the issue of drift is nonexistent.

This paper is organized as follows. First, the theory behind moving masses as an attitude control mechanism will be discussed. Next, the dynamics governing the motion of the satellite platform will be developed and a control strategy will be introduced. The experimental configuration and data collection/reduction will then be summarized. The results of the experiment will then be presented.

**Development of Theory**

The underactuated rotational dynamics of a 3U cubesat have been developed in [1]. In this formulation, the cubesat center of gravity (CG) is not constant and the two masses are configured in an orthogonal arrangement as shown in figure 1. This mass configuration provides a large amount of control freedom but it is only possible to test such a system in an environment free of gravity. By ‘sweeping out area’ in actuator space, these masses are able to create a net rotation about the axis perpendicular to the plane in which the actuators are moving. In a laboratory environment in which gravity is present, the center of gravity and the center of mass must remain constant. When testing moving masses on the Whorl-I simulator, any shift in the center of gravity would cause the platform to tilt and attempt to re-stabilize about its’ new center of gravity. This creates difficulties in testing complete three dimensional pointing control algorithms on a laboratory spherical air bearing.

![Figure 1: 3U cubesat with two moving masses in an orthogonal configuration](image-url)
Dynamics

In this section, the formulation of [1] is altered and extended to account for a constant CG as well as a parallel mass configuration. Euler’s equations of rigid body dynamics describe the effect that external torques have on the angular momentum of a rigid body. This law states that the time rate of change of angular momentum is equal to the external torques on a rigid body, or

$$\dot{\vec{H}} = \vec{M}$$

(1)

where the vector $\vec{M}$ is an external torque vector. The time rate of change of the angular velocity, with no external torques, of a rigid body is obtainable directly from Euler’s equations and is written as

$$\dot{\vec{\omega}}_{B/N} = -I^{-1}_B \left[ I_B \vec{\omega}_{B/N} + \vec{\omega}_{B/N} \times (I_B \vec{\omega}_{B/N}) \right]$$

(2)

The subscript $B$ refers to a coordinate frame which is fixed to the spacecraft simulator as shown in figure 3. The subscript $N$ refers to a coordinate frame fixed in inertial space. In general, spacecraft are controlled by exerting an external torque by means of thrusters or momentum exchange devices. This causes the right hand side of equation 1 to be non-zero and equal to the control torque. These methods of control also generally assume that the inertia of the rigid body is constant. This causes the $I_B$ in equation 2 to be zero. The use of moving mass actuators allows the inertia to be varied which in turn varies the angular velocity of the spacecraft.

An expression which describes the time rate of change of the inertia due to the moving masses is given by

$$\dot{I}_B = 2m \left[ 2\vec{r}^T \vec{v} [1] - \vec{r} \vec{v}^T - \vec{v} \vec{r}^T \right]$$

(3)

Here, $m$ is the mass being moved by each actuator and the vectors $\vec{r}$ and $\vec{v}$ are the position and velocity, respectively, of the masses relative to the body fixed coordinate frame. The term $[1]$ represents the identity matrix. The actuator configuration represented in figure 2 will change the inertia about the $z$ and $y$ axes only. Because the simulator has full freedom about the $z$ axis, the actuators will be used to regulate the angular velocity about this axis. The time rate of change of the angular velocity about the $z$ axis can be solved for analytically by inserting the position and velocity vectors into equation 2. The position and velocity vectors are given by

$$\vec{r}_1 = -\vec{r}_2 = [x, 0, 0]^T$$
$$\vec{v}_1 = -\vec{v}_2 = [\dot{x}, 0, 0]^T$$

Therefore, it is found that the angular acceleration about the $z$ axis is given by

$$\frac{d\omega_z}{dt} = -\frac{4m\omega_z \dot{x}}{1 + 2m x^2}$$

(4)

This equation provides straightforward insight into how the position and velocity of the moving masses affect the angular velocity of the simulator. By inspection of equation 4, it is apparent that moving the masses out a sufficient distance $x'$ will cause the angular velocity to be driven to zero. This of course is only physically limited by the length of the actuators being used and the size of the spacecraft. It is important to note that this equation corresponds to pure rotation about the yaw axis. In reality, angular velocity components are likely to exist about other body fixed axes. The analytic expressions for these other angular velocity components are found similarly to the component about the yaw axis. For the purpose of testing in the lab, the only component being considered is the $z$ component due to the full freedom of the Whorl-I simulator about this axis.

Because the dynamics of the simulator must be relative to an inertial frame, the
body fixed axes may be mapped to the the inertial frame through the following standard transformation given in [2].

\[
\dot{R}^N_B = -\omega^x_{B/N} R_{B/N}
\]

(5)

where \( R_{B/N} \) denotes the rotation matrix mapping the body frame to the inertial frame. This equation is numerically integrated using MATLAB to provide the instantaneous orientation of the simulator at any time during the simulation.

To perform accurate numerical simulations of these dynamics, it is necessary to determine the inertia properties. In a paper published by a graduate student at Virginia Tech, Scott Kowalchuk, the inertia matrix of the Whorl-I spacecraft simulator was estimated from detailed CAD drawings. The inertia matrix was estimated to be

\[
I = \begin{bmatrix}
8.8325 & 0.6854 & -0.1579 \\
0.6854 & 7.6989 & -0.1035 \\
-0.1579 & -0.1035 & 12.9964
\end{bmatrix} \text{kg-m}^2
\]

Controller

As a preliminary attempt at controlling the nonlinear underactuated dynamics of a rigid body with moving masses, an open loop controller was developed. Open loop commands were sent directly to the masses and the response of the system was measured using the IMU. The control input \( u \) contains the positions of the masses along each of their respective actuator tracks. The control input is then defined as

\[
u = [r_{m1} \ r_{m2}]^T
\]

This simple open loop controller has been used to validate numerical models of the dynamics of a rigid satellite model with two moving masses. The advantage of testing this system on the Whorl-I spacecraft simulator is that the simulator has full freedom about its yaw, or \( z \), axis (see figure 3). In future work, a feedback control system will be developed in order to automate the process data collection/analysis and actuation commands.

Experiment Configuration

The Whorl-I spacecraft simulator with a defined body-fixed coordinate system is shown in figure 3. The simulator sits on a spherical air bearing which is able to simulate a space environment using pressurized air. The platform is equipped with reaction wheels, a variable speed control moment gyro, a Newtonian damper, several linear actuators, and inertial measurement units. Referring to the coordinate system defined on the simulator, rotation about the \( x \) axis is defined as pitch, rotation about the \( y \) axis is defined as roll, and rotation about the \( z \) axis is defined as yaw. The simulator has full freedom in yaw and \( \pm 5^\circ \) in pitch and roll. Because the simulator only has full freedom in yaw, it is the object of the experiment to regulate the angular velocity and control pointing about the \( z \) axis. Newmark Systems MSL linear stage actuators with MDrive Intelligent steppers motors are fastened to Whorl-I and used to translate the masses. The actuators are
Figure 3: Whorl-I Spacecraft Simulator in the Virginia Tech Space Systems Simulation Laboratory

Figure 4: Newmark Systems MSL Linear Stage Actuators

shown in figure 4. These actuators are arranged such that they do not affect the CG of the simulator. The CG must remain constant and fixed in the center of the simulator in order to avoid any pendulum dynamic effects. Pendulum dynamics occur due to an offset off the instantaneous center of mass. This offset creates an external torque due to the gravity vector acting through the instantaneous center of mass.

A VectorNAV VN-100 inertial measurement unit (IMU) is used to collect and process the angular velocity data in real time. The IMU is capable of measuring magnetic field, angular rates, angular displacements, direction cosine matrices, quaternions, etc. The IMU will be configured to measure angular rate data about each of axes outlined in figure 3. The VectorNAV IMU is shown in figure 5. Each actuator and the IMU are securely fastened to the Whorl-I spacecraft simulator to avoid any external error in orientation measurement. MATLAB code has been developed to interface the IMU and the actuators. The code reads orientation data from the IMU and uses a scaling factor to send the actuators position commands. A basic diagram of the experimental configuration is outlined in figure 2.

Results

In this section, the results of the numerical simulation and the experimental data are outlined. The effects of differently weighted masses are discussed.

Numerical Simulations

The numerical simulations of the spacecraft platform with linearly translating masses were performed using MATLAB and a numerical integration scheme. The state of the system was modeled as a 3x1 vector defined by

$$s = [\omega_z \ x \ \dot{x}]^T$$  \hspace{1cm} (6)
\( \omega_z \) is the \( z \) axis angular velocity, \( x \) is the actuator position, and \( \dot{x} \) is the actuator velocity. MATLAB was used to integrate the derivative of equation 6 in order to obtain a time history of the angular velocity about the \( z \) axis. The numerical integration employs a fourth order Runge-Kutta method with variable step sizes. The initial conditions were set to

\[
\omega_z(t_0) = \frac{\pi}{6} \text{ rad/s}
\]
\[
x(t_0) = 0 \text{ in}
\]
\[
\dot{x}(t_0) = 3 \text{ in/s}
\]

The weight of the masses varied between 5 and 15 kg. The results of the simulation are outlined in figure 6. Moving at a rate of three inches/second, as defined in the initial conditions, the masses move from their initial positions to their fully extended positions in roughly two seconds. Therefore, the abscissa spans from \( t=0 \) seconds to \( t=2 \) seconds.

Figure 6: Results of the numerical simulation. Angular velocity about the \( z \) axis can be seen to decrease as the masses are extended from their initial positions.

It can be seen from the figure that in a matter of only two seconds, the 15 kg masses are capable of decreasing the \( z \) angular velocity by roughly 0.1 rad/s or nearly 6 deg/s. The ability of these moving masses to regulate angular velocity provides a novel means of satellite attitude control. An application of this concept which will be considered in future work is that of a constant Earth pointing satellite. Because no mass has discarded from the satellite, the original angular velocity may be recovered by simply moving the masses back in to their original positions. This is a similar concept to the yo-yo de-spin strategy for de-spinning a satellite with the advantage that no extra mass needs to be dumped from the satellite.

An interesting observation made during the numerical simulations of this system is that extending masses to a sufficient distance will drive the angular velocity to very near zero(yo-yo de-spin technique). If long slender rods, for example, were used, it would be possible to obtain much larger angular velocity reductions. If such a technique was used and the rods were extended for a longer time period, the angular velocity may be reduced by nearly an order of magnitude. These results are shown in figure 7.

Figure 7: Angular velocity regulation using slender rods as opposed to masses.
Experimental Results
As described in the experiment configuration section, two linear actuators were fastened to the Whorl-I spacecraft simulator. The simulator was given an initial angular velocity of roughly 0.25-0.27 rad/s. During data collection, the simulator was free of external torques. Position commands were sent to the actuators from a MATLAB program in order to extend them from their initial positions to their final positions (see figure 2). The IMU was then used to collect data from gyro readings. This process was repeated three separate times in order to eliminate as many uncertainties as possible. The results of the three experimental tests are outlined in figure 8. The initial angular velocity can be seen to vary among the separate iterations. This is due to human error but it is irrelevant to the overall trend of the angular velocity profiles. In each iteration, the masses remained at their initial positions for the initial three to four seconds of rotation. At this time the masses were extended. As can be seen in the plot, the angular velocity decreases by 0.2-0.25 rad/s. This trend agrees with the numerical predictions, although the magnitude of the angular velocity reduction is lower. The existence of this difference is due to the actuator mass to total mass ratio. The numerical simulations used 5, 10, and 15 kg masses on a 50 kg satellite. The masses used in the experiment were 7.5 kg compared to a platform mass of over 100 kg. In an attempt to obtain more agreeable results between the numerical simulations and the experimental data, heavier weights were used on the actuators but it was found that the actuators were incapable of moving larger weights.

Observing figures 8 and 9, it can be seen that the angular velocity begins to decrease when the mass is extended from its’ initial position at \( t=3 \) s.

Figure 8: Angular velocity readings from the IMU during the experiment. As seen in the figure, the angular velocity about the z axis is seen to decrease when the masses are extended away from the C.G.

Figure 9: Absolute actuator position during the experiment. Actuator remains in initial position until time=3 s.

Conclusions
This research has focused on using moving masses as a method of attitude control and momentum management for spinning spacecraft. Numerical simulations were performed
and experimental data was collected to validate the use of moving masses in the aforementioned applications. It has been determined that moving mass motion control is a novel and efficient means of control. Important factors to consider when using this type of control are actuator mass to total mass ratios, actuator lengths, actuator speeds, and the types (i.e. rectangular masses, rods, etc.) of masses being translated.

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References


