

Preliminary Studies on the Development of a 3-D Space Radiation Transport Model

Introduction

Future missions in Earth's orbit, to near-Earth space, the Moon and Mars will expose crewmembers to the space radiation environment for extended periods of time. The high energy protons in the Earth's trapped radiation belts and from solar particle events (SPE), and the high energy atomic nuclei (HZE) in the galactic cosmic rays (GCR) have the potential of producing serious radiation health effects in astronauts and can damage sensitive equipment that is critical to mission success. The intensity of these ions must therefore be reduced while holding secondary radiation to a minimum within the spacecraft interior where the astronauts spend most of their time. Effective shielding materials are therefore being sought to mitigate the effects of such radiation without a large negative impact on mission costs. It is clearly impractical to verify the shielding properties of every candidate material and configuration in space. For this reason, shield designers rely heavily upon models of radiation transport and measurements taken at particle accelerators, which play an important role in the model design and validation process.

The insufficient understanding of HZE particle induced risk to astronauts and their equipment presents mission planners with a basic challenge. In order to estimate this risk, it is necessary to do shield material studies and to understand the distribution of particle flux both in deep space and in planetary environments. It is also necessary to understand the nuclear interaction processes that take place in shielding materials, such as fragmentation and secondary particle production that are functions of cross sections exhibiting angular and energy dependence. This requires knowledge of material transmission characteristics either through laboratory testing or improved theoretical modeling. In consequence, Old Dominion University faculty and scientists at NASA Langley Research Center are currently involved in a collaborative effort to develop mathematical and computational methods to improve or replace existing radiation transport codes.

According to a recent National Research Council Report (Committee on the Evaluation of Radiation Shielding for Space Exploration, 2008), predictions derived from radiation transport calculations need to be tested using a common code for laboratory and space measurements that have been validated with accelerator results. However, as noted by Wilson et al. (1990), numerical solution methods for the Boltzmann transport equation are best suited to space radiations where the energy spectra are smooth over large energy intervals, and less suited to the simulation of laboratory beams which exhibit large spectral variation over a very limited energy domain and large energy derivative. Since HZETRN, the current state-of-the-art code for space radiation transport, is based on such methods, it is not readily validated by comparison with laboratory experiments. This is a weakness that needs to be addressed and can be resolved using a Green's function approach; a primary focus of the proposed research. A second weakness of the current code is its inability to address nuclear interaction processes, such as fragmentation and secondary particle production, which exhibit angular dependence. The current code is one-dimensional and treats such processes by using various approximations that speed up and greatly simplify transport calculations. Since three-dimensional effects could be significant for neutrons, low energy target fragments, and evaporation particles, it has been proposed that a fully three-dimensional Green's function transport code, capable of simulating high charge and energy ions in both space and laboratory environments, be developed.

In this project, emphasis will be placed on the construction and approximation the first

perturbation term in the Neumann series solution of the Boltzmann Transport Equation. A number of approximations will be considered and comparisons made between them in order to determine the most efficient. In addition, the investigator will learn how existing codes work in order to make comparisons between new and existing solutions with a view to examining the significance of three dimensional effects.

The Boltzmann Equation

Consideration is given to the transport of high charge and energy (HZE) ions through a three-dimensional convex region V that is bounded by a smooth surface ∂V and is filled with a shielding material.

As shown in Fig. 1, \mathbf{x} and \mathbf{x}_b are the position vectors of arbitrary points in V and ∂V respectively, $\mathbf{n}(\mathbf{x}_b)$ is the unit outward normal at \mathbf{x}_b , and $\boldsymbol{\Omega}$ is an arbitrary unit vector. The symbol $B(\mathbf{x})$ denotes the bundle of unit vectors at \mathbf{x} and $B(\mathbf{x}_b)$ the bundle of unit vectors at \mathbf{x}_b that satisfy the constraint $\boldsymbol{\Omega} \cdot \mathbf{n}(\mathbf{x}_b) < 0$. In addition, the symbol $\mathbf{x}'(\mathbf{x}, \boldsymbol{\Omega})$ is used to denote the point where the ray through \mathbf{x} in the direction $\boldsymbol{\Omega}$ first meets the boundary ∂V .

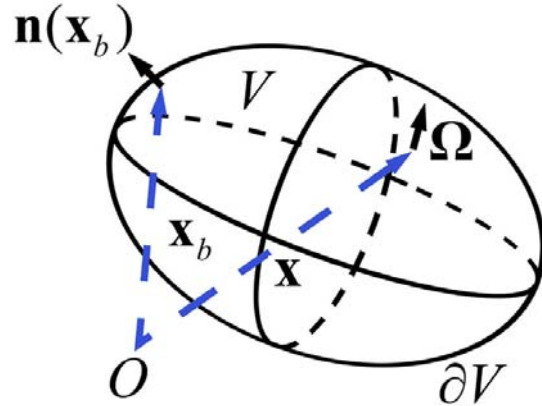


Fig. 1

The transport of HZE ions through V is governed by the linear Boltzmann equation for the flux density $\phi_j(\mathbf{x}, \boldsymbol{\Omega}, E)$ of type j particles and is given by Wilson et al. (1991a) as

$$\boldsymbol{\Omega} \cdot \nabla \phi_j(\mathbf{x}, \boldsymbol{\Omega}, E) + \sigma_j(E) \phi_j(\mathbf{x}, \boldsymbol{\Omega}, E) = \sum_k \int \sigma_{jk}(\boldsymbol{\Omega}, \boldsymbol{\Omega}', E, E') \phi_k(\mathbf{x}, \boldsymbol{\Omega}', E') d\boldsymbol{\Omega}' dE', \quad j=1, 2, \dots, N, \quad (1)$$

where N is the number of particle types, and $\sigma_j(E)$ and $\sigma_{jk}(\boldsymbol{\Omega}, \boldsymbol{\Omega}', E, E')$ are the media macroscopic cross sections. The differential cross sections $\sigma_{jk}(\boldsymbol{\Omega}, \boldsymbol{\Omega}', E, E')$ represent all those processes by which type k particles moving in direction $\boldsymbol{\Omega}'$ with energy E' produce a type j particle in direction $\boldsymbol{\Omega}$ with energy E . Note that there may be several reactions which produce a particular product, and the appropriate cross sections for equation (1) are the inclusive ones. The total cross section $\sigma_j(E)$ with the medium for each particle type of energy E may be expanded as

$$\sigma_j(E) = \sigma_j^{at}(E) + \sigma_j^{el}(E) + \sigma_j^r(E) \quad (2)$$

Where the first term refers to collision with atomic electrons, the second term is for elastic nuclear scattering, and the third term describes nuclear reactions. The differential cross sections may be expanded in a similar fashion. Many atomic collisions ($\sim 10^6$) occur in a centimeter of

ordinary matter, whereas $\sim 10^3$ nuclear Coulomb elastic collisions occur per centimeter, while nuclear reactions are separated by a fraction to many centimeters depending on energy and particle type. This ordering allows flexibility in expanding solutions to the Boltzmann equation as a sequence of physical perturbative approximations. We are required to solve equation (1) subject to boundary conditions of the type $\phi_j(\mathbf{x}_b, \mathbf{\Omega}, E) = F_j(\mathbf{x}_b, \mathbf{\Omega}, E)$, $\mathbf{x}_b \in \partial V$, $\mathbf{\Omega} \in B(\mathbf{x}_b)$. In the case of a unit source at the boundary, $F_j(\mathbf{x}_b, \mathbf{\Omega}, E)$ takes the special form

$$F_j(\mathbf{x}_b, \mathbf{\Omega}, E) = \frac{\delta_{jm}}{2\pi} \delta(1 - \mathbf{\Omega} \cdot \mathbf{\Omega}_0) \delta(E - E_0) \bar{\delta}(\mathbf{x}_b - \mathbf{x}_0), \quad \mathbf{x}_b \in \partial V, \mathbf{\Omega} \in B(\mathbf{x}_b) \quad (3)$$

and the corresponding solution, which is called the Green's function, is denoted by the symbol $G_{jm}[\mathbf{x}, \mathbf{x}_0, \mathbf{\Omega}, \mathbf{\Omega}_0, E, E_0]$. Once the Green's function is known, the solution for arbitrary boundary conditions $F_j(\mathbf{x}_b, \mathbf{\Omega}, E)$ is then given by

$$\varphi_j(\mathbf{x}, \mathbf{\Omega}, E) = \sum_{m \geq j} \int_{\partial V} d\mathbf{x}_0 \int_{4\pi} d\mathbf{\Omega}_0 \int_E^\infty dE_0 G_{jm}(\mathbf{x}, \mathbf{x}_0, \mathbf{\Omega}, \mathbf{\Omega}_0, E, E_0) F_m(\mathbf{x}_0, \mathbf{\Omega}_0, E_0) \quad (4)$$

Since space radiation boundary conditions are obtained from spaceflight measurements and are broad functions of energy for each ion type, equation (4) is typically implemented by numerical integration procedures.

Solution Methods

The transport code HZETRN currently used by NASA is based on a solution of equation (1) that is developed by a marching procedure (Wilson et al., 1991). Such numerical solution methods for the Boltzmann transport equation are best suited to space radiation where the energy spectra are smooth over large energy intervals and less suited to the simulation of laboratory beams, which exhibit large spectral variation over a very limited energy domain and large energy derivative. As a result, space codes based on these numerical methods are not readily validated by comparison with laboratory experiments (Wilson et al., 1991). Only analytical procedures are able to simulate both space radiation and laboratory beam transport with equal ability in a common procedure, and to that end the new code GRNTRN is currently under development.

In order to develop the solution further, we write Eq. (1) in operator notation by defining the vector array field function $\Phi = [\phi_j(\mathbf{x}, \mathbf{\Omega}, E)]$, the drift operator $\mathbf{D} = [\mathbf{\Omega} \cdot \nabla]$, and the interaction operator $\mathbf{I} = \Xi - \sigma = \left[\sum_j \int \sigma_{jk}(\mathbf{\Omega}, \mathbf{\Omega}', E, E') d\mathbf{\Omega}' dE' - \sigma_j(E) \right]$, with the understanding that \mathbf{I} has three parts associated with atomic, elastic, and reactive processes as given in Eq. (2). Eq. (1) is then written as

$$[\mathbf{D} - \mathbf{I}^{at} - \mathbf{I}^{el} + \sigma^r] \cdot \Phi = \Xi^r \cdot \Phi, \quad (5)$$

or equivalently as the Volterra integral equation

$$\Phi^{for} = \mathbf{G}^0 \cdot \Phi_B + \mathbf{Q} \cdot \mathbf{L} \cdot \Xi^r \cdot \Phi \quad (6)$$

where Φ_B is the appropriate boundary condition, Q is a quadrature operator representing integration along a ray in the Ω direction from the boundary point $x'(x, \Omega)$ to the current position x , G^0 is the solution of the homogeneous form of Equation (5) with a unit source at the boundary, and L is a linear operator closely related to G^0 . G^0 is called the zero order Green's function and is associated with the primary flux.

Since (6) is a Volterra integral equation, it admits a Neumann series solution

$$\begin{aligned}\Phi &= [G^0 + (Q \cdot L \cdot \Xi^r) \cdot G^0 + (Q \cdot L \cdot \Xi^r)^2 \cdot G^0 + (Q \cdot L \cdot \Xi^r)^3 \cdot G^0 + \dots] \cdot \Phi_B \\ &= [G^0 + G^1 + G^2 + G^3 + \dots] \cdot \Phi_B\end{aligned}\quad (7)$$

that can be evaluated directly or prescribed as a marching procedure in either a perturbative sense as in the current form of HZETRN or a nonperturbative sense as in GRNTRN (Wilson et al., 1994a, Tweed et al., 2005).

The above formalism lends the following interpretation of the solution. The operator G^0 propagates the particles with attenuation processes. The first term, $G^0 \cdot \Phi_B$, propagates primary ions from a boundary point x' to the interior point x . The expression $\Xi^r \cdot G^0 \cdot \Phi_B$ is the production density of first generation of fragments at position x on the line segment $[x', x]$. These particles are propagated to the interior by $L \cdot \Xi^r \cdot G^0 \cdot \Phi_B$. Lastly,

$G^1 \cdot \Phi_B = (Q \cdot L \cdot \Xi^r) \cdot G^0 \cdot \Phi_B$ represents the sum of all the first generation fragments being propagated to x from sources in the line segment $[x', x]$ and so on. Once G^0 and L have been identified, the remaining terms in the Neumann series (7) may be found via the recurrence formula

$$G^n = [Q \cdot L \cdot \Xi^r] \cdot G^{n-1}, \quad n \geq 1. \quad (8)$$

Construction of the Neumann series for the one dimensional problem has been discussed in detail by Tweed et al. (2005) where accurate analytical approximations are obtained for the first three terms, and the remainder of the series is estimated by a non-perturbative technique. Similar techniques can be used to develop the three dimensional solution.

Objective

NASA's current transport code HZETRN will ultimately be replaced by the new Green's function code, GRNTRN. GRNTRN will incorporate much more nuclear physics, will be fully three-dimensional and be more accurate. It will contain improved elastic scattering transport procedures. It will also address energy loss due to straggling, nuclear attenuation, and nuclear fragmentation with energy dispersion and downshifts. As a first step towards developing GRNTRN, it will be necessary to construct and approximate the zero order Green's function G^0 . A number of approximations will be considered and comparisons made between them in order to determine the most efficient. In addition, the investigator will learn how existing codes work in order to make comparisons between new and existing solutions with a view to examining the

significance of three-dimensional effects. Lastly, some validation studies will be conducted by comparing theoretical predictions with existing experimental data.

Proposed Timeline

The investigator will intern at NASA Langley Research Center for one day a week during the academic year and for ten weeks during the summer. The proposed work schedule is as follows:

1. Read and study project background materials including recent publications on HZETRN and the one-dimensional version of GRNTRN. (2 months).
2. Learn to use the HZETRN and 1-D GRNTRN codes (3 months).
3. Assist in the development and of the 3-D Green's function model (2 months).
4. Code and test the zero order Green's function and compare approximations (2 months).
5. Make comparisons between the 1-D and 3-D codes (1 month).
6. Conduct validation studies (1 months).
7. Analyze the results, draw conclusions and write up a final report. (1 month).

Conclusion

The proposed study is part of the first phase in the development of a fully three-dimensional Green's function space radiation transport model. It is expected to shed some light on the significance of three-dimensional effects by making comparisons with one-dimensional code predictions and experimental data. It is also expected to help identify those approximation techniques that are most efficient for this and later phases of the 3-D GRNTRN modeling initiative. The project will provide the investigator with the opportunity to learn how existing transport codes work and to acquire some of the skills necessary to participate in the modeling process and future code development efforts.

References

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