THE SOLVABILITY OF THE RADIOISTY EQUATION ON UNOCCLUDED CURVES

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Abstract

The radiosity equation is a mathematical equation modeling the radiosity, or brightness, at points on a surface in terms of the reflectivity and emissivity at these points. In this report we study the mapping and spectral properties of the radiosity operator on $L^p(\partial\Omega)$ spaces, $p \in (1, \infty)$, where $\partial\Omega$ is an unoccluded curve. Specifically, we consider the case where $\partial\Omega$ is an infinite sector of aperture $\theta \in (0, 2\pi)$ in $\mathbb{R}^2$ and the reflectivity function is piecewise constant. We show that the solvability of the radiosity equation hinges on whether or not the point 1 lies in the spectrum of the naturally associated integral operator.

1. Introduction

The radiosity, or brightness, of a surface is defined as the rate at which energy leaves the surface and it is given in terms of energy per unit time per unit area. It incorporates both energy emitted by the surface (emissivity) and the energy reflected from other surfaces (reflectivity). The radiosity equation is an integral equation modeling the radiosity at points on a surface in terms of the emissivity and reflectivity at these points. It is relevant to the Global Illumination Problem in computer graphics and has applications to thermal radiation in mechanical engineering. The radiosity equation takes the following form. Let $\partial\Omega \subset \mathbb{R}^2$ be the underlying surface and $P \in \partial\Omega$. Then

$$E(P) =$$

$$u(P) - \rho(P) \int_{\partial\Omega} u(Q) G(P,Q) V(P,Q) d\sigma(Q),$$

where $u(P)$ is the unknown radiosity, $\rho(P)$ is the reflectivity function, V is the "line-of-sight" function, and $E(P)$ is the emissivity function. Since most realistic surfaces do not reflect all of the light that they receive, the function ρ is assumed to be in the interval $[0, 1)$. The function V takes two possible values, $V = 1$ in the unoccluded case (i.e., when all points on the surface can "see" each other along a straight line segment which does not intersect with $\partial\Omega$), and $V = 0$ otherwise.

For $P \in \partial\Omega$ we associate the radiosity equation above with the operator

$$\begin{aligned} &K : L^p(\partial\Omega) \to L^p(\partial\Omega), \\ &Ku(P) := \int_{\partial\Omega} u(Q) G(P,Q) d\sigma(Q), \end{aligned} \tag{1.1}$$

and call this the radiosity operator. With this notation, we now write the radiosity equation as

$$E = u - Ku,$$

or, equivalently,

$$(I - K)u = E, \tag{1.2}$$

where K is defined in (1.1) and I the identity operator. Then solving (1.2) reduces to finding when the operator

$$(I - K)^{-1} : L^p(\partial\Omega) \to L^p(\partial\Omega)$$

is meaningful on the given function space. In this report the function spaces under consideration are the classical Lebesgue spaces of p-integrable functions on the boundary, denoted $L^p(\partial\Omega)$, $1 < p < \infty$.

In order to determine the invertibility of $(I - K)$ on $L^p(\partial\Omega)$, we study the mapping and spectral properties of the radiosity operator on $L^p(\partial\Omega)$ spaces, $p \in (1, \infty)$, where $\partial\Omega$ is an unoccluded curve (i.e. $V \equiv 1$). Specifically, we consider the case where $\partial\Omega$ is an infinite sector of aperture $\theta \in (0, 2\pi)$ in $\mathbb{R}^2$ and the reflectivity function is constant on each ray of the sector. We also assume that $\partial\Omega$ is a *Lambertian diffuse reflector*, meaning that the reflectivity at any point P on $\partial\Omega$ is uniform in all directions from P. This assumption is physically true for many surfaces, with two notable exceptions being mirrors or glossy facades.

Under these conditions, we utilize Mellin transform techniques to analyze the spectrum. In fact, we show that the solvability of the integral equation hinges on whether or not the point 1 lies in the spectrum of the radiosity operator. We also obtain estimates on the spectral radius of the operator which are useful in numerical treatments of the problem.

Our main results in this direction are the following two theorems.

Theorem 1.1. *The operator* $(I - K) : L^p(\partial\Omega) \to L^p(\partial\Omega)$ *is invertible for all* $p \in (1, \infty)$ *when* $\theta = \frac{\pi}{2}$. *Further, the spectral radius of* K *as in (1.1) is bounded by*

$$\tfrac{3}{4\sqrt{2}} \approx 0.53.$$

Theorem 1.2. *The spectral radius of* $K : L^p(\partial\Omega) \to L^p(\partial\Omega)$ *as in (1.1) is less than 1 for all* $1 < p < \infty$ *provided that* $\partial\Omega$ *is an infinite sector of aperture*

$$\theta > 2\cos^{-1}\left(\sqrt{\tfrac{2}{\pi}}\right) \approx 1.29.$$

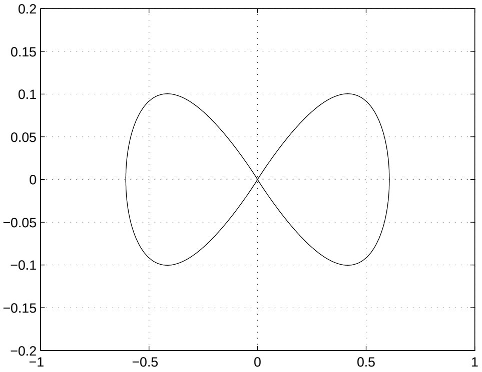

Figure 1: The L^6 spectrum of the radiosity operator on a sector of aperture $\theta = \frac{2\pi}{5}$.

The paper proceeds as follows. Section 2 contains preliminaries and definitions relating to $L^p(\partial\Omega)$ spaces, the spectrum of an operator and Mellin transform techniques. Section 3 provides a brief overview of the

Gamma function and several of its special properties which will be useful for us in the remainder of the report. In Section 4 we compute the spectrum of the radiosity operator. Then in Section 5 we look at the special case of $\partial\Omega$ forming a ninety degree angle, i.e., $\theta = \frac{\pi}{2}$. In this situation we also obtain estimates on the spectral radius of the radiosity operator which are relevant to the numerical treatment of the radiosity equation. Finally, Section 6 contains analysis of the spectrum of the radiosity operator for a spectrum of arbitrary aperture $\theta \in (0, \pi)$.

2. Preliminaries

Let $\Omega \subset \mathbb{R}^2$ be a domain with boundary $\partial\Omega$.

Definition 2.1. *The Lebesgue space of p-integrable functions, denoted* $L^p(\partial\Omega)$*, with* $1 \leq p < \infty$ *is defined as*

$$L^p(\partial\Omega) := \left\{ f;\ \left(\int_{\partial\Omega} |f|^p d\sigma \right)^{1/p} < \infty \right\},$$

where $d\sigma$ *stands for the surface measure of* $\partial\Omega$*.*

Now recall that a function $\phi : \mathbb{R} \to \mathbb{R}$ is called Lipschitz if there exists $\omega > 0$ such that

$$|\phi(x') - \phi(y')| \leq \omega |x' - y'|,$$

for all x', y' in the domain of ϕ. Then we define the following.

Definition 2.2. *A domain* $\Omega \subset \mathbb{R}^2$ *lying above the graph of a Lipschitz function* $\phi : \mathbb{R} \to \mathbb{R}$ *is called a special Lipschitz domain. That is*

$$\Omega := \{(x_1, x_2) \in \mathbb{R}^2 :\ x_2 > \phi(x_1)\}.$$

In this work we consider Ω to be the interior of an infinite angle of aperture $\theta \in (0, 2\pi)$, which is in the class of special Lipschitz domains.

Next we will outline properties of the Mellin transform as well as results regarding spectral analysis via Mellin transform techniques (see e.g. [8], [9], and [10]).

Definition 2.3. *Let* $f \in \mathcal{C}_0^\infty(\mathbb{R}_+)$*, the space of infinitely differentiable functions with compact support on the interval* $[0, \infty)$*. Then the Mellin transform of* f *is defined as*

$$\mathcal{M}f(z) := \int_0^\infty x^{z-1} f(z) dx, \quad z \in \mathbb{C}.$$

Before going further with regard to the Mellin transform, we introduce the spectrum of an operator and define the spectral radius.

Definition 2.4. *Let* $\mathcal{X}$ *be a Banach space and let* $T : \mathcal{X} \to \mathcal{X}$ *be linear and continuous. The spectrum of* T*, denoted by* $\sigma(T; \mathcal{X})$*, is the set*

$$\sigma(T; \mathcal{X}) := \{w \in \mathbb{C} : wI - T \text{ is not invertible on } \mathcal{X}\},$$

where I *denotes the identity operator.*

Definition 2.5. *The spectral radius of the operator* T *on a Banach space* $\mathcal{X}$ *is given by*

$$\rho(T; \mathcal{X}) := \max\{|w| : w \in \sigma(T; \mathcal{X})\}.$$

Note that the spectral radius of T corresponds to the radius of the smallest closed circular disc centered at the origin containing $\sigma(T; \mathcal{X})$.

To conclude this section, we define Hardy kernel operators and discuss their spectra.

Definition 2.6. *Let* $k(x,y)$ *be a measurable function on* $\mathbb{R}_+ \times \mathbb{R}_+$*. The* k *is a Hardy kernel for* $L^p(\mathbb{R}_+)$*,* $1 \leq p < \infty$ *provided that*

1. $k(x,y)$ *is homogeneous of degree -1, meaning for any* $\lambda > 0$ *we have that* $k(\lambda x, \lambda y) = \lambda^{-1} k(x,y)$*.*
2. $\int_0^\infty |k(1,y)| y^{-1/p} dy < \infty$.

Further, a matrix $k = (k_{ij})_{i,j=1,2}$ *of measurable functions on* $\mathbb{R}_+ \times \mathbb{R}_+$ *is called a Hardy kernel for* $L^p(\mathbb{R}_+)$ *provided that each entry* k_{ij} *in the matrix is a Hardy kernel.*

Definition 2.7. *Let* $k = (k_{ij})_{i,j=1,2}$ *be a Hardy kernel for* $L^p(\mathbb{R}_+)$ *and let* $\vec{f} = (f_1, f_2)$ *be a vector-valued function in* $L^p(\mathbb{R}_+)$*,* $1 \leq p < \infty$*. This means that* $f_i \in L^p(\mathbb{R}_+)$ *for* $i,j = 1,2$ *and* $1 \leq p < \infty$*. For any* $\vec{f} \in L^p(\mathbb{R}_+)$*, define the Hardy kernel operator,* $\mathcal{K}$*, as follows*

$$\mathcal{K}\vec{f}(x) := \int_0^\infty k(x,y)\vec{f}(y)dy, \tag{2.1}$$

for $x \in \mathbb{R}_+$*.*

Theorem 2.8. *If* k *is a Hardy kernel for* $L^p(\mathbb{R}_+)$*,* $1 \leq p < \infty$*, then the operator* $\mathcal{K}$ *defined in (2.1) is a bounded operator on* $L^p(\mathbb{R}_+)$*. The spectrum of* $\mathcal{K}$ *as an operator on* $L^p(\mathbb{R}_+)$ *consists of the closure in the plane of all points* $w \in \mathbb{C}$ *such that*

$$\begin{gathered} det(wI - \mathcal{M}k)(\tfrac{1}{p} + i\xi) = 0, \\ \text{for some } \xi \in \mathbb{R}, \end{gathered}$$

where I *is the identity matrix operator and* $\mathcal{M}k := (\mathcal{M}k_{ij})_{i,j=1,2}$*.*

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For the remainder of the report we will refer to the matrix $\mathcal{M}k$ as the matrix of Mellin symbols of the operator $\mathcal{K}$ on $L^p(\mathbb{R}_+)$, $1 \leq p < \infty$.

3. The Gamma Function

In the upcoming analysis a special function known as the Gamma function, denoted $\Gamma(\cdot)$, will play a predominant role. We begin here with a brief review of several of its basic properties which we will appeal to in the subsequent sections of this work.

Definition 3.1. *For* $\operatorname{Re} z > 0$ *the Gamma function is defined as*

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt. \tag{3.1}$$

Notice that $\Gamma(1) = 1$, and via integration by parts on (3.1), $\Gamma(2) = 1$ as well. In fact, the following addition formula holds true,

$$\Gamma(z+1) = z\Gamma(z), \tag{3.2}$$

for any $\operatorname{Re} z > 0$. Further, one sees that (3.2) implies that for n a positive integer, $\Gamma(n+1) = n!$. Thus the Gamma function generalizes the factorial function for non-integer and complex values.

Values of $\Gamma(z)$ for $\operatorname{Re} z < 0$ can be deduced from values of $\Gamma(z)$ for $\operatorname{Re} z > 0$. The following formula is derived from a known result on an infinite product. It states that for $\operatorname{Re} z > 0$,

$$\Gamma(z)\Gamma(-z) = -\tfrac{\pi}{z}\csc(\pi z).$$

We can now deduce the following formulas for $z \in \mathbb{C}$ which will be useful in the sequel (see e.g., p. 11-12 in [11]).

- Multiplication formulas

$$\Gamma(z)\Gamma(1-z) = \pi\csc(\pi z),$$
$$\Gamma(\tfrac{1}{2}+z)\Gamma(\tfrac{1}{2}-z) = \pi\sec(\pi z).$$

- Duplication formula

$$\Gamma(2z) = \frac{2^{2z-1}\Gamma(z)\Gamma(z+\frac{1}{2})}{\sqrt{\pi}}.$$

One additional and useful fact can be derived by computing the multiplication formula in (3.3), with $z = \frac{1}{2}$. This yields

$$\begin{aligned}\Gamma(\tfrac{1}{2})\Gamma(1-\tfrac{1}{2}) &= \pi\csc\tfrac{\pi}{2} \\ &= \pi. \end{aligned} \qquad (3.3)$$

In other words, $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

4. The Mellin symbol of K

In this section we find an explicit expression for the spectrum of the radiosity operator K acting on $L^p(\partial\Omega)$ spaces, $1 < p < \infty$. Consider the case where $\partial\Omega$ is an infinite angle of aperture $\theta \in (0, 2\pi)$. The upcoming presentation remains valid in the case $\theta = 2\pi - \theta$ so we can restrict our analysis to $\theta \in (0,\pi)$. Denote by $(\partial\Omega)_1$ the left-hand ray of the infinite angle and by $(\partial\Omega)_2$ the right-hand ray. Let the reflectivity function ρ be piecewise constant on each ray, that is $\rho|_{(\partial\Omega)_1} \equiv \rho_1$ and $\rho|_{(\partial\Omega)_2} \equiv \rho_2$, with the additional assumption that $\rho_1, \rho_2 \in [0,1)$. Recall the radiosity equation introduced in (1.2),

$$(I-K)u = E,$$

with K defined in (1.1). We can solve for the unknown radiosity,

$$u = (I-K)^{-1}E, \qquad (4.1)$$

provided that 1 is not an element of $\sigma(K; L^p(\partial\Omega))$. In other words, the exclusion of 1 from the spectrum of $K : L^p(\partial\Omega) \to L^p(\partial\Omega)$ means that $(I-K)$ is invertible. Subsequently, we undergo a careful analysis of the $L^p(\partial\Omega)$ spectrum of the radiosity operator K on unoccluded curves. Indeed, if we can prove that $\rho(K; L^p(\partial\Omega)) < 1$ we are done.

To begin this analysis, set the kernel of K to be

$$k(P,Q) := \rho(P)G(P,Q).$$

In the case where $\partial\Omega$ is a connected Lipschitz curve, the function $G(P,Q)$ is known, yielding the kernel

$$k(P,Q) = \rho(P)\frac{\langle (P-Q), \nu(P)\rangle\langle (Q-P), \nu(Q)\rangle}{2|P-Q|^3}. \qquad (4.2)$$

where ν is the outward unit normal vector on $\partial\Omega$ and $\langle\cdot,\cdot\rangle$ stands for the usual dot product in two dimensions.

It can be shown that (4.2) satisfies the criteria of Hardy kernel and thus the spectrum of the radiosity operator can be studied via the Mellin transform techniques of Theorem 2.8.

Recall that $\partial\Omega$ is an infinite angle aperture $\theta \in (0,\pi)$ and P, Q lie on $\partial\Omega$. Let $s := |P|$ and $t := |Q|$. The following two lemmas compute the kernel $k(s,t)$, considering all possible cases for the location of P, Q on $\partial\Omega$.

Lemma 4.1. *If P, Q lie on opposite rays of $\partial\Omega$, then*

$$k(s,t) = \rho(P)\frac{st\sin^2\theta}{2(s^2 - 2st\cos\theta + t^2)^{3/2}},$$

for $P \in (\partial\Omega)$.

Lemma 4.2. *If P, Q lie on the same side of $\partial\Omega$, then*

$$k(s,t) = 0.$$

Corollary 4.3. *The kernel $k(s,t)$ in general is given by*

$$k(s,t) = \frac{st\sin^2\theta}{2(s^2 - 2st\cos\theta + t^2)^{3/2}} \left(\begin{array}{cc} 0 & \rho_1 \\ \rho_2 & 0 \end{array} \right).$$

Setting $u = \frac{s}{t}$, we choose to describe $k(s,t)$ in terms of one variable as follows,

$$k(u) = k(u,1) = \frac{u\sin^2\theta}{2(u^2 - 2u\cos\theta + 1)^{3/2}} \left(\begin{array}{cc} 0 & \rho_1 \\ \rho_2 & 0 \end{array} \right).$$

Lemma 4.4. *Given an angle $\theta \in (0,\pi)$, the essential spectrum of the operator K on $L^p(\partial\Omega)$ consists of the two parametric curves $w_1(\frac{1}{p} + i\,\cdot\,)$ and $w_2(\frac{1}{p} + i\,\cdot\,)$ where*

$$\begin{aligned} w_1 &= \sqrt{\rho_1\rho_2}\,\frac{\sin^2\theta}{2}\mathcal{M}f(z+1), \\ w_2 &= -\sqrt{\rho_1\rho_2}\,\frac{\sin^2\theta}{2}\mathcal{M}f(z+1). \end{aligned} \tag{4.3}$$

Here, $z = \frac{1}{p} + iy, 1 < p < \infty, y \in \mathbb{R}$ and $\mathcal{M}f(z+1)$ stands for the Mellin transform of the function

$$f(u) := \frac{1}{(u^2 + 2u\cos\gamma + 1)^{3/2}},$$

evaluated at $z+1$ and with $\gamma := \pi - \theta$.

We have found that the spectrum of K acting on $L^p(\partial\Omega)$ consists of the two parametric curves $w_1(\frac{1}{p}+i\,\cdot\,)$ and $w_2(\frac{1}{p}+i\,\cdot\,)$ given in (4.3). Our next step is to further analyze the spectrum of K via Mellin transform tools. Specifically, equation 2.58 on p. 24 in [14], with $\nu = \frac{3}{2}$ gives

$$\mathcal{M}f(z+1) = \frac{2}{\sin\theta}\beta(z+1, 2-z)P^{-1}_{z-1}(-\cos\theta).$$

The equation above holds for

$$0 < \mathrm{Re}\, z + 1 < 2\mathrm{Re}\,\nu.$$

We meet this condition since

$$\mathrm{Re}\, z + 1 = \tfrac{1}{p} + 1 < 2 \text{ and } 2\mathrm{Re}\,\nu = 3.$$

Then the Mellin symbol of K acting on $L^p(\partial\Omega)$ is given by the equation

$$\mathcal{M}k(z) = \sin\theta\beta(z+1, 2-z)P^{-1}_{z-1}(-\cos\theta). \tag{4.4}$$

In (4.4), β is the Euler beta function. Studied by Euler and Legendre, and given its name by J. Binet, this special function is defined as (see e.g., equation (1) on p. 15 in [11])

$$\beta(a,b) := \int_0^1 t^{a-1}(1-t)^{b-1}dt,$$

where $\mathrm{Re}\, a > 0$ and $\mathrm{Re}\, b > 0$. In our case,

$$\begin{aligned} a = z+1 \quad &\Rightarrow \quad \mathrm{Re}\, a = \tfrac{1}{p} + 1 > 0, \\ b = 2-z \quad &\Rightarrow \quad \mathrm{Re}\, b = 2 - \tfrac{1}{p} > 0. \end{aligned}$$

The Beta function can also be expressed in terms of the Gamma function, as given in equation (3) on p. 15 of [11],

$$\beta(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$

In terms of (4.4), this definition yields the following

$$\beta(z+1, 2-z) \quad = \quad \frac{\Gamma(z+1)\Gamma(2-z)}{2}.$$

Next in (4.4), P is called a Legendre Function of the first kind (see e.g., 6.2.3 on p. 211 in [11]), with $\mu = -1, \nu = z - 1$ and $z = -\cos\theta$. In general, a Legendre function $P_\nu^\mu(z)$ converges for $|z| < 1$.

In the instance of (4.4) it takes the form

$$P_{z-1}^{-1}(-\cos\theta) =$$

$$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}\; {}_2F_1\left(1-z, z; 2; \frac{1}{2} + \frac{1}{2}\cos\theta\right).$$

Finally, directly above we have the hypergeometric function ${}_2F_1$ (see also p. 9 and equation (13) on p. 39 of [11]). In general it is defined as

$$ {}_2F_1(a, b; c; z) := \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \qquad (4.5)$$

where $(a)_n := a(a+1)(a+2)\ldots(a+n-1)$ is called the rising factorial. In our case (4.5) translates into

$$ {}_2F_1\left(1-z, z; 2; \frac{1}{2} + \frac{1}{2}\cos\theta\right) =$$

$$\frac{\Gamma(2)}{\Gamma(z)\Gamma(1-z)} \cdot \Sigma,$$

where $\Sigma := \sum_{n=1}^{\infty}$

$$\frac{\Gamma(1-z+n)\Gamma(z+n)}{\Gamma(2+n)} \frac{(\frac{1}{2} + \frac{1}{2}\cos\theta)^n}{n!}.$$

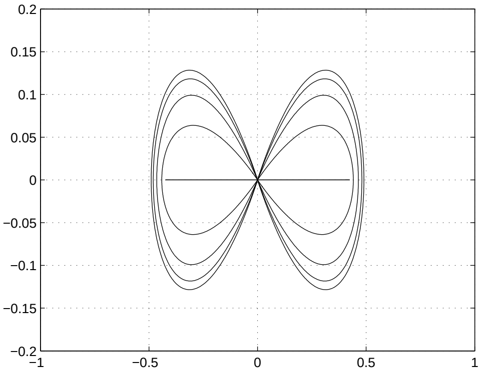

Figure 2: The $L^p(\partial\Omega)$ spectrum of the radiosity operator on a sector of aperture $\theta = \frac{\pi}{2}$ for $p = 2, 4, 8, 16$, and 32.

5. Estimates for $\theta = \frac{\pi}{2}$

Now we consider the scenario where $\theta = \frac{\pi}{2}$ and utilize cancelations that take place in the special functions defined in Section 4. More specifically, in the case of the first quadrant we take advantage of special properties of the hypergeometric function ${}_2F_1$ to obtain an explicit bound on the spectral radius of K on $L^p(\partial\Omega)$. While we omit the majority of the calculations, note that in this case the argument of the hypergeometric function reduces to

$$ {}_2F_1\left(1-z, z; 2; \frac{1}{2}\right),$$

which is known to equal

$$\frac{\sqrt{\pi}}{2\Gamma(\frac{3-z}{2})\Gamma(\frac{2+z}{2})}, \qquad (5.1)$$

where $\Gamma(\cdot)$ is as defined in Section 3 and $z = \frac{1}{p} + iy$, $1 < p < \infty$ and $y \in \mathbb{R}$. Using this identity, along with special properties of the Gamma function, we arrive at the first of our main results.

Theorem 5.1. *The operator* $(I-K)$ *acting on* $L^p(\partial\Omega)$ *into* $L^p(\partial\Omega)$ *is invertible for all* $p \in (1,\infty)$ *when* $\theta = \frac{\pi}{2}$*. Further,*

$$\rho(K; L^p(\partial\Omega)) \leq \frac{3}{4\sqrt{2}} \approx 0.53. \qquad (5.2)$$

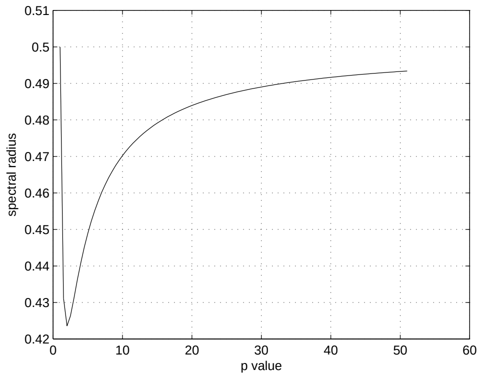

Figure 3: The spectral radius of $K : L^p(\partial\Omega) \to L^p(\partial\Omega)$ for a sector of aperture $\theta = \frac{\pi}{2}$ and $p = 1, \ldots, 50$.

The estimate in (5.2) is relevant in the numerical treatment of (1.2), in particular in establishing the convergence of collocation and graded mesh methods.

6. Estimates for general $\theta \in (0, 2\pi)$

In this section we study the Mellin symbol of the radiosity equation for an angle of arbitrary aperture $\theta \in (0, 2\pi)$. We can no longer take advantage of the special identity on the hypergeometric function given in (5.1), so we rely instead on estimates. The main result in this more general case is as follows.

Theorem 6.1. *The spectral radius of* $K : L^p(\partial\Omega) \to L^p(\partial\Omega)$ *as in (1.1), is less than 1 for all* $1 < p < \infty$ *provided that* $\partial\Omega$ *is an infinite sector of aperture*

$$\theta > 2\cos^{-1}\left(\sqrt{\frac{2}{\pi}}\right) \approx 1.29. \qquad (6.1)$$

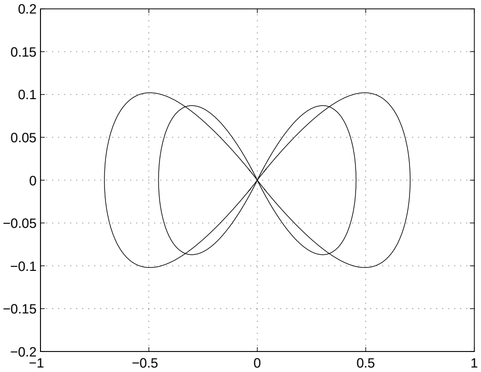

Figure 4: The L^6 spectrum of the radiosity operator on sectors of aperture $\theta = \frac{\pi}{2}$ and $\theta = \frac{\pi}{3}$.

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