MODELING AND VIBRATION SUPPRESSION OF A PRESSURIZED OPTICAL MEMBRANE

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Abstract
Optical membranes are being pursued for their ability to replace the conventional mirrors that are used to correct wave front aberration and space based telescopes. Among some of the many benefits of using optical membranes, is their ability to considerably reduce the weight of the structure. As a secondary effect, the cost of transportation, which is of great interest in space applications, is reduced as well. Another interesting advantage is the ability to have a continuous surface for the attenuation of wave front aberrations, instead of a discrete grid of rigid mirrors that have to be individually controlled. The effect of adding a pressurized cavity behind an optical membrane are examined in this paper by coupling an acoustic cylindrical cavity to a cylindrical membrane at its boundaries. This paper also looks at using a positive position feedback controller for vibration suppression of the membrane. This is done by using a centralized acoustic source in the cavity as the method of actuation. The acoustic actuation is of great interest since it does not mass load the membrane in the conventional way, as most methods of actuation would.

Introduction
With the move to space inflatable structures, which are a promising technology that can potentially revolutionize the design of large satellite systems, the need to replace the optical rigid mirrors with lightweight mirrors is desired. For many large deployable space structure applications, space inflatable structures have many advantages over mechanically deployed systems. Some of these advantages include being lighter weight, having a higher packaging efficiency, a lower life cycle cost, lower parts counts, and higher deployment reliability. Advantages like these have sparked a great interest in these inflatable structures and also the replacement of rigid mirrors for optical lightweight membranes.

These thin film membranes offer order of magnitude size increase for aperture and weight reduction in lightweight space structures (Moore 2002). Other advantages are low cost, reproducibility, easily stored and deployed. With the interest in these lightweight reflective membranes the need for proper dynamical models and surface control are needed (Moore 2002). There is extensive work on the control of rigid optical mirrors, as in (Schrader 1999), but little work has been done on the control of reflective membranes for the use of telescope aperture or for the correction of wavefront aberrations.

As satellites are retargeted in a slew maneuver their ability to acquire information suffers due to the vibrations induced in this movement. The satellite vibrations need to settle before the image capturing can continue. It is essential that the satellite recover as fast as possible, for the obvious reason, after such a maneuver. Thus, controlling the vibrations of the membrane mirror will reduce the down time of the satellite. For these reasons it is of interest to study the effect of adding a pressurized cavity to the back of an optical membrane. Once the dynamically coupling effects are understood it is of interest to use this cavity for vibration suppression through acoustic actuation.

Previous works have looked at the coupling of similar systems but none have looked at modeling this type of system for the purpose of optical thin film membranes and acoustic actuation. In (Dowel 1962), the effect of a rectangular cavity on panel vibration was examined but was restricted to a one-way coupling and only symmetric modes were considered. In (Rajalingham 1998), they examined the coupling between an open-ended cavity and a circular membrane in which the
cavity did not experience a feedback response from the membrane. Gorman 2001, studied the response of a pressure sensor design with a cavity/disc-plate coupling. Similarly, in (Chen 2006) they used a plate diaphragm coupling, however, they only considered axis symmetric coupling. In (Kato 2007), one panel of a rectangular chamber was used to drive the opposing panel. Axis symmetry and plate models were used in this case as well.

This paper examines the system coupling between an acoustic cavity and a thin film membrane and the possibilities of using acoustic actuation for the vibration control of a thin membrane.

The modeling for the system will be split into two cases. First, modeling of the cavity/membrane structure due to an acoustic excitation in the cavity will be discussed. Second, the same system will be evaluated due to a disturbance inflicted on the membrane without the acoustic excitation.

Membrane/Cavity Acoustic Actuation Model

As mentioned before the approach for modeling will start by looking at the individual subsystems before coupling them together. The equation for the free vibration of a membrane is given by (Inman 2007) as

$$\tau \nabla^2 \mathbf{w} (r, \theta, z, t) = \rho \frac{\partial^2 \mathbf{w} (r, \theta, t)}{\partial t^2}$$

(1)

where $\tau$ is tension per unit length and $\rho$ is density per unit area.

For the cavity we start with the 3D wave equation as expressed in (Fahy 1998).

$$\nabla^2 p (r, \theta, z, t) = \frac{1}{c_o^2} \frac{\partial^2 p (r, \theta, z, t)}{\partial t^2}$$

(2)

where $c_o$ is the acoustical velocity. For both equations, the solution for the membrane displacement and cavity pressure can be assumed to be of the form

$$\mathbf{w} = \sum_{n=1}^{N} \phi_n (r, \theta) b_n$$

(3)

$$p = \sum_{m=1}^{M} \psi_m (r, \theta, z) a_m$$

(4)

respectively. Where $\phi_n$ and $b_n$ are the mode shapes and modal amplitude of the membrane and $\psi_m$ and $a_m$ are the mode shapes and modal amplitude of the cavity. Equations (3) and (4) are taken to be in the frequency domain.

Coupling of the system starts by discretizing the surface at the interface. Each point is spaced such that the areas are equal for each location. Once the interface is discretized the contribution from one subsystem to the other can be defined. Figure 2 shows a schematic of the discretization as well as the subsystems and how they contribute dynamically to each other. In the figure, at the interface point $(\theta, r, z)$, $v_i$ is the velocity of the membrane, $p_i$ is the pressure of

![Figure 1. Subsystem drawing of the optical membrane cavity structure with a piston actuator for the acoustic excitation.](image-url)
the cavity at the area $s_i$ around the discretized $i^{th}$ point between the membrane and the pressure cavity. For the actuation, $v_f$ is the actuation velocity and $s_f$ is the actuation area at the $f^{th}$ point. The velocity of the membrane can be obtained by multiplying equation (3) by $j\omega$.

Figure 2. The model is discretized and the interface is analyzed for the coupling between the subsystems.

Figure 2 shows in more detail how each subsystem contributes to each other. In the case of the membrane, it exhibits a dynamical contribution from the interaction with the cavity. When examining the cavity, the cavity exhibits a dynamical contribution from both the membrane at one end and from the acoustic actuation at the other end. These contributions are expressed in equations (5) and (6) by adding them to the modal amplitudes of each individual subsystem.

Substituting equations (5) and (6) into equations (3) and (4) in placing them in matrix form yields

$$P = \Psi_p^T A_p \left[ \Psi_p S V + \Psi_{pf} S_f V_f \right]$$

$$V = -\Phi_s^T A_s \Phi_s S P$$

where $\Psi_p$ is the matrix of discretized pressure mode shapes with dimension $(m \times i)$ and $\Phi_s$ is the matrix of the discretized membrane mode shapes with dimension $(n \times i)$. The matrices $S$, $S_f$, $A_p$, and $A_s$ are diagonal matrices with their corresponding area and modal amplitudes respectively. The unknown pressure vector $P$ and $V$ have dimensions $(i,1)$. Decoupling equations (7) and (8) by substitution results in the pressure and velocity equations for the coupled system

$$P = \left[ I + \Psi_p^T A_s \Psi_p \Phi_s^T A_s \Phi_s \right]^{-1} \Psi_p^T A_p \Psi_p S_f V_f$$

$$V = -\Phi_s^T A_s \Phi_s S P$$

In equations (9) and (10) $A_p$ and $A_s$ are frequency dependent and this system need to be solved for each frequency of interest.

To validate this model analytically, it is useful to look at different scenarios and observe how the system behaves.

For the first case the tension in the membrane is varied. In theory, the more the membrane is tensioned, the coupled model should tend to a solid cavity. In Figure 3, this is seen for various tension scenarios.
red line represents the pressure inside a solid cavity. The black, blue green and magenta lines represent 8 lbs, 20 lbs, 50 lbs and 100 lbs of tension on the membrane.

The frequency response function (FRF) of the membrane as the cavity depth is changed can also be analyzed. Figure 4 shows how the cavity modes couple into the system as the cavity depth is increased. As the cavity depth is increased, the modes of the cavity are lowered and play a more dominant role in the lower frequency bandwidth.

Figure 4. The membrane FRF (on the center point) is affected by the change in cavity depth. The Cyan, red, green and blue lines represent a cavity depth of 0.1, 0.06, 0.03, and 0.001 meters.

The effects brought forth by the addition of a cavity on the membrane, can be observed by comparing the dynamics of a single membrane to the same pressure loads obtained by the acoustic actuator in the coupled system. The same can be done for the cavity. Figure 5 illustrates the differences in these cases. Looking at the membrane FRF, the peak at the 300Hz is a contribution from the coupling of both subsystems that is not observed in the single membrane FRF (dashed line). The same result is evident in the cavity FRF.

Figure 5. On top: FRF at the center point of the membrane of the coupled system (solid line) and a single membrane subjected to the same pressure source of excitation (dashed line). Bottom: FRF of the cavity of the coupled system (solid line) and a solid cavity subjected to the same acoustic excitation (dashed line). These results are for a cavity depth of 0.2 meters.

Figure 6 shows that as the cavity depth is decreased, the dominant modes of the cavity shift to a higher frequency bandwidth and the lower frequency dynamics are attenuated.

From these results, it can be seen that the cavity depth is an important parameter in the overall performance of the coupled system. Depending on the application, careful consideration needs to be taken when deciding on a cavity depth.
Figure 6. On top: FRF at the center point of the membrane of the coupled system (solid line) and a single membrane subjected to the same pressure source of excitation (dashed line). Bottom: FRF of the cavity of the coupled system (solid line) and a solid cavity subjected to the same acoustic excitation (dashed line). These results are for a cavity depth of 0.03 meters.

Membrane/Cavity Disturbance Model

For the disturbance model, the assumption is that the disturbance inflicted on the system comes from an external force on the membrane. As compared to the previous model there is no acoustic actuation in this model. Figure 7 shows the schematics of the system under consideration.

Figure 7. The model is discretized and the interface is analyzed for the coupling between the subsystems. In this case the actuation comes from a disturbance to the membrane.

The main difference for the disturbance model lies in adequately changing the modal amplitudes to reflect the different contributions from each subsystem.

The new modal amplitudes for the membrane and cavity are

\[ b_n = A_n(\omega) \left[ \sum s_i \phi_i(r, \theta) - \sum s_j D_{ij} \phi_j(r, \theta) \right] \]  

\[ a_n = A_n(\omega) \left[ \sum \nu \psi_j(r, \theta) \right] \]

where \( D_{ij} \) represents the external disturbance pressure applied to the membrane. Following the same procedure as before by placing these equations in matrix form and solving for the velocity and pressure results in

\[ V_o = \left[ I + \Phi^T A_i \Phi_i S \right]^{-1} \Phi^T A_i \Phi_i S D \]  

\[ P_o = -\nu^T A_i \Psi_i S \left[ I + \Phi^T A_i \Phi_i S \right]^{-1} \Phi^T A_i \Phi_i S D \]
As before $A_p$ and $A_v$ are frequency dependent and this system need to be solved for each frequency of interest. The transpose of modal coupling matrix appears here as well.

Figure 8 and Figure 9 show two responses for a cavity depth of 0.2 meters and 0.03 meters. The figures also have the individual uncoupled cases subjected to the same excitation for comparison. Looking at both depth cases, the same effects as seen in the previous model can be seen here as well. Increasing the cavity depth lowers the cavity modes and increases their contribution on the lower frequency bandwidth.

![Figure 8](image1.png)

**Figure 8.** On top: FRF at the center point of the membrane of the coupled system (solid line) and a single membrane subjected to the same disturbance on the membrane (dashed line). Bottom: FRF of the cavity of the coupled system (solid line) and a solid cavity subjected to the same disturbance pressure (dashed line). These results are for a cavity depth of 0.2 meters.

![Figure 9](image2.png)

**Figure 9.** On top: FRF at the center point of the membrane of the coupled system (solid line) and a single membrane subjected to the same disturbance on the membrane (dashed line). Bottom: FRF of the cavity of the coupled system (solid line) and a solid cavity subjected to the same disturbance pressure (dashed line). These results are for a cavity depth of 0.03 meters.

The cavity depth can be used as a design parameter when coupling a membrane and cavity together. When choosing a cavity depth, specific performance criteria needs to be examined since looking at the actuation versus disturbance model may not lead to the same cavity depth requirement. From an acoustic actuation model, a larger cavity depth is of interest due to the fact that a higher control authority may be obtained at lower frequencies. From a disturbance model, a large cavity couples the two subsystems strongly and generates a higher resonance magnitude at lower frequencies.

**Control**

Control of the optical membrane surface is of great interest since the optics cannot be used while the membrane vibrates above a certain tolerance. Optical membrane systems with a diameter greater than a few centimeters will require some form of active control (Patrick 2005). The purpose of the acoustic actuation model is of interest for the purpose of controlling...
the membrane surface. For the control case, the problem schematic can be defined using Figure 10. Our coupled system has an input \( U \) from the acoustic piston a disturbance \( W \) on the membrane and both outputs \( z \) and \( y \) are at the membrane.

![Figure 10. Schematics of the input/output relationship in membrane/cavity coupled system](image)

The inputs and outputs of this system are related by

\[
\begin{align*}
\text{z} &= \mathcal{H}_{zw} \text{w} + \mathcal{H}_{zu} \text{u} \\
\text{y} &= \mathcal{H}_{yw} \text{w} + \mathcal{H}_{yu} \text{u}
\end{align*}
\]  

where \( y \) is the velocity (V) out at the membrane due to the acoustic actuation and \( z \) is the velocity (\( V_D \)) due to the disturbance force.

For the controller, the \( y \) output will be used as feedback to the compensator in this system. Figure 11 demonstrates the schematics of the closed loop system.

![Figure 11. Schematics of the feedback control.](image)

Combining equations (13) and (14) for the closed loop system result in the closed loop equation of the system

\[
\text{z} = [\mathcal{H}_{zw} + \mathcal{H}_{zu} \mathcal{K} \{1 - \mathcal{H}_{yu} \mathcal{K}\}^{-1} \mathcal{H}_{yw}] \text{w} \tag{15}
\]

where \( \mathcal{K} \) is the feedback compensator of the closed loop system. There are many approaches for choosing. In our case a positive position feedback (Goh 1985, Caughey 1995, Tarazaga 2007) approach will be used to define \( \mathcal{K} \).

In the frequency domain the controller can be can be expressed as

\[
\mathcal{K}(j\omega) = \frac{g \omega_f}{(j\omega)^3 + 2\zeta_f \omega_f (j\omega) + \omega_f^2}
\]  

where \( g \) is the controller gain and \( \omega_f \) and \( \zeta_f \) are the natural frequency and damping ratio of the controller respectively.

Figure 12 shows a comparison between the open loop (blue line) and closed loop (red line) response of the system when trying to dampen out the first mode.

![Figure 12. Closed loop (red line) and open loop (blue line) response of the system when trying to dampen the first mode.](image)

Figure 13 shows the same scenario as experienced at different locations on the membrane. In both cases, the effect of the controller can be seen affecting higher modes. This is because the controller in this case does not die out quickly enough before reaching the other modes.

![Figure 13. Closed loop (red line) and open loop (blue line) response of the system when trying to dampen the first mode as seen from different locations on the membrane.](image)
Conclusion

This paper demonstrates feasibility of using a mobility modeling technique to couple an acoustic cavity and an optical membrane for dynamics studies. The positive position feedback controller worked well in using the acoustic actuation for controlling of the first mode.

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References


