Abstract

Rhythmic movements associated with animal locomotion are controlled by neuronal circuits known as central pattern generators (CPG). These biological control systems appear to entrain to the natural frequencies of the mechanical systems they control, taking advantage of the resonance of the structure, resulting in efficient control. The ultimate goal is employing these controls in a biomimetic autonomous underwater vehicle so as to capture, and possibly improve upon, the performance capabilities of animals like the manta ray. To this end, this paper investigates the CPG control of a simple tensegrity structure using a reciprocal inhibition oscillator (RIO). The dynamics of a tensegrity structure are linearized about a nominal configuration, and a synthesized RIO is used as the control input. The method of harmonic balance is used to verify the robustness of entrainment. Simulations are used to verify entrainment.

1 Introduction

Rhythmic motion in animals is ultimately controlled by central pattern generators. CPGs are a system of neurons connected in such a way that their outputs autonomously oscillate with particular phase relationships between individual neurons. Brown first illustrated that the central pattern generator was responsible for this behavior and suggested that sensory feedback served to regulate the CPGs activity. He also cited reciprocal inhibition, where the activity of antagonistic neurons inhibit each other, as a fundamental unit in rhythm generation. Delcomyn argued the case of central control in a study covering diverse rhythmic activities in over 40 species using isolation, deafferentation, and paralysis techniques. This study conclusively illustrated that the intrinsic properties of the central nervous system are “capable of providing the proper timing of muscle activation.” Pearce and Friesen illustrated the contribution of sensory feedback, as well as peripheral neuronal and mechanical effects, to the coordination of rhythmic swimming activity by comparing in situ medicinal leeches and those with isolated nerve chords. Friesen then illustrated that reciprocal inhibition is responsible for oscillatory motion in many species. He also showed that dynamic properties like synaptic fatigue and impulse adaptation play an important role in oscillatory circuits. Yu et al. illustrated the role of sensory feedback in leech swimming by comparing in situ and severed nerve chord specimens, observing that sensory feedback was capable of coordinating swimming. Ultimately, the literature illustrates that central pattern generators, consisting of neurons with reciprocal inhibition, are responsible for rhythmic patterns. Sensory feedback has the ability to fine tune these patterns, resulting in coordinated signals essential to animal locomotion.

These studies in biology have attracted the attention of engineers seeking efficient control methodologies in robotics. Taga et al. employed neuronal circuits to control double pendulums modeling legs for bipedal locomotion, and observed robust entrainment of the CPG to the mechanical system’s resonance frequency. Williamson used a CPG to control an arm with entrainment to a hanging mass system, as well as turning a crank and ‘playing’ with a slinky. Kimura et al. experimentally verified synthetic neuronal control of a quadruped walking on irregular terrain as well as running on flat terrain using reflex signals to coordinate the CPG. Lewis and Bekey and Nakanishi et al. also applied neuronal circuits to gait control and Lewis et al. demonstrated the ability to create in silico CPGs. Chen and Iwasaki developed a method for synthesizing oscillators with prescribed frequency, phase, and amplitude, and how to design such oscillators for feedback control of an optimized locomotion variable.
Engineers have also investigated the conditions and mechanisms that provide entrainment of neuronal circuits to dynamic systems. Verdaasdonk et al. simulated the neuro-musculo-skeletal system of the human arm, observing the conditions necessary for a reciprocal inhibition oscillator (RIO) to entrain to the musculo-skeletal system. Williams and DeWeerth performed a case study of a simulated single degree of freedom mass-spring-damper system controlled by an RIO, and commented on the form of feedback necessary for the RIO to entrain to the dynamic system when the RIO’s intrinsic frequency is either above or below the resonance of the mechanical system. Iwasaki and Zheng compared the entrainment of different RIO models to a pendulum, mapping entrainment against neuronal parameters. Then they used the method of harmonic balance to predict entrainment. Futakata and Iwasaki also used the method of harmonic balance to analyze resonance entrainment of an RIO to a pendulum. This extensive analysis illustrated that negative integral feedback and positive rate feedback are responsible for robust entrainment.

The systems discussed above all utilize classical structures and models like the pendulum and mass-spring-damper. This study proposes the use of tensegrity structures as a robotic morphing locomotor. Tensegrities are systems of bars held in compression by a network of cables in tension and are statically and dynamically nonlinear and indeterminant. Originally developed by Snelson and Fuller, tensegrities have attracted interest due to their high strength to weight ratios as well as their actuation capabilities. The statics and dynamics of tensegrities have been studied extensively. Their use as deployable antennae has been analyzed, as well as controllable platforms for flight simulators. Both the cables and bars provide a method of actuation and sensing, making tensegrities a great candidate for CPG control.

This study will present the tools necessary to analyze the coupled system of a CPG controlled tensegrity. It will begin with the dynamics of tensegrities (Sec. 2) and the architecture of a reciprocal inhibition oscillator (Sec. 3). The entrainment of the controller to the structure and its robustness will then be analyzed using the method of harmonic balance, illustrating the potential capabilities of a tensegrity based locomotor.

2 Dynamics of a Tensegrity Structure

2.1 Equations of Motion

Tensegrity structures are systems of bars held in compression by cables in tension. The tensegrity system in this study is a Type 2 structure (i.e. the maximum number of bars sharing the same node is two) and is shown in Fig. 1. We consider the cables to be rigid with uniformly distributed mass and are connected through a rotational joint with linear kinetic damping. All bar lengths and masses are assumed equivalent. The cables are assumed massless and generate tension linear elastically when stretched beyond their manufacturing length, and their elastic moduli are equivalent. We will use the cables to actuate the system, and will model the actuation as a change in manufacturing length.

![Figure 1: Type 2, 3 cell tensegrity: bars shown in black, strings in grey](image)

Constraints are workless, holonomic, schleronomic, and bilateral. In this study, we will neglect outside forces. With these assumptions, and using a Lagrangian formulation, the nonlinear equations of motion take the form:

\[
M(\theta)\ddot{\theta} + c(\theta, \dot{\theta}) + C(\theta)\dot{\theta} + A(\theta)\tau(\theta) = 0,
\]

where \(\theta = [\theta_1, \ldots, \theta_6]^T \in \mathbb{R}^6\) and the angles are measured with respect to the horizontal axis of the inertial frame. \(M(\theta)\) and \(C(\theta)\) represent the inertia and damping matrices respectively, and the inertia matrix is given in the Appendix for reference. \(A(\theta)\tau(\theta)\) represents the vector of torques generated by the elastic forces resulting from the difference between current (stretched) length \(l_i\) and the manufacturing (unstretched) length \(l_{m,i}\) of the cable \(s_i\). These lengths can be grouped into vectors \(l, l_m \in \mathbb{R}^9\) respectively. The vector \(c(\theta, \dot{\theta})\) represents the quadratic elements arising from coupling of the degrees of freedom. This system of equations can be linearized...
about a nominal configuration \( \theta_e \), resulting in:

\[
M_e \ddot{\theta} + C_e \dot{\theta} + K_e \theta + B_e u = 0,
\]

\( M_e, C_e, K_e \in \mathbb{R}^{6 \times 6} \),

\[ M_e := M(\theta_e), C_e := C(\theta_e), \]

\[ K_e[i, j] := \frac{\partial^2 U}{\partial \theta_i \partial \theta_j} \bigg|_{\theta_e}, \quad (2) \]

\[ B_e \in \mathbb{R}^{6 \times 9}, B_e[i, j] := \frac{\partial^2 U}{\partial \theta_i \partial l_{e_i}} \bigg|_{\theta_e}, \]

\[ \vartheta := \theta - \theta_e, \]

\[ u := l_o - l_{e_i}, \]

where \( M_e, C_e, K_e \) and \( B_e \) are the linearized mass, damping, stiffness, and control matrices and \( U \) represents the potential energy of the system. Let \( l_{e_i} \) and \( l_o \) represent the nominal manufacturing length and nominal stretched length vectors respectively. It is also relevant to note that the control input is simply the change in manufacturing length of the cables, allowing easy access to the control variables in a physical structure by simply pulling on the cables.

The nominal configuration used in the linearization, \( \theta_e \), must have a valid state of prestress and is therefore not arbitrary. Using the principle of virtual work, we can derive an analytic condition for the state of prestress in our symmetric tensegrity structure:

\[
\frac{w}{2l_b} < \sin(\theta_{o}) < \frac{l_b^2 + w^2}{2lw},
\]

\( T_i = T_V, i = 1...6, \)

\( T_7 = 2T_9 = T_{H1}, T_8 = T_{H2}, \)

\( \theta_1 = \theta_3 = \theta_5 = \theta_o, \)

\( \theta_2 = \theta_4 = \theta_6 = 2\pi - \theta_o, \)

where

\[
[T_V T_{H1} T_{H2}] := P \left[ T_{ov} T_{oh} T_{oh2} \right],
\]

\[
[T_{ov} T_{oh} T_{oh2}] := \begin{bmatrix} T_V T_{H1} T_{H2} \end{bmatrix},
\]

\[ T_V := 1, \]

\[ T_{H1} := \frac{2w}{\sqrt{l_b^2 + w^2 - 2l_b w \sin(\theta_o)}}, \]

\[ T_{H2} := \frac{2(2l_b \sin(\theta_o) - w)}{\sqrt{l_b^2 + w^2 - 2l_b w \sin(\theta_o)}}, \]

and \( T_i \) and \( l_b \) are the tension in the \( i^{th} \) string and bar length respectively. Note that \( T_i \) is calculated using a prestress factor \( P \). This is, in essence, a scaling factor that sets the tension in the cables, increasing or decreasing the natural frequencies of the structure 32, 39, 40. Our desired structure has \( \sin(\theta_o) = l_b/w \), as illustrated in Fig. 1. This is the nominal configuration about which the structure will oscillate when controlled by a CPG.

### 2.2 Dynamic Characteristics

With a linearized dynamic model complete, we can analyze the mass and stiffness matrices to shed light on the characteristic (natural) frequencies of the structure. Calculating the eigenvector-eigenvalue pairs of \( M_e^{-1} K_e \) results in the associated mode shapes and square of natural frequencies of the structure. In the case of our structure with six degrees of freedom, we obtain the six expected modes of natural frequency show in Fig. 2.

![Figure 2: Modes of vibration (rad/s)](image)

(a) \( \omega_n = 5.6 \)

(b) \( \omega_n = 39.7 \)

(c) \( \omega_n = 48.1 \)

(d) \( \omega_n = 91.0 \)

(e) \( \omega_n = 96.2 \)

(f) \( \omega_n = 134.2 \)

The first mode, which looks similar to the first mode of a beam in bending, will be our targeted resonant mode. Whether or not a CPG can tune to this mode to exploit resonance will dictate a successful integration.

It is also important to check that our linear dynamic model is accurately predicting the behavior of the nonlinear tensegrity system. Verification of linearized systems has been carried out before 39, but this study only looked at the decaying oscillations of the system. We must make sure that resonant control of the linear and nonlinear systems agree. To do
so, each system is simulated with a control input at resonance, oscillating the first and forth strings at 180° phase offset. Figure 3 shows strong agreement between the two models.

2.3 State Space Model

When we integrate our control system to our tensegrity structure (Sec. 4), it will be useful to have a state space model of our system. We will be controlling the manufacturing length of the strings, and using the current length as the feedback signal. However, the current length is a nonlinear function of the generalized coordinates. For this reason, we must perform another linearization. This is calculated by taking the gradient of the current length vector with respect to the generalized coordinates, and evaluating it at the equilibrium position.

\[ \tilde{l} = \nabla l|_{\theta_e} \theta \]  

(4)

Our state space model can now be constructed as follows:

\[
\dot{x} = Ax + Bu \\
y = Cx, 
\]

(5)

where

\[ x = \begin{bmatrix} \dot{\theta} \\ \dot{\theta} \end{bmatrix}^T \]

\[ A = \begin{bmatrix} 0_6 & I_6 \\ -M_e^{-1}K_e & -M_e^{-1}C_e \end{bmatrix} \]

\[ B = \begin{bmatrix} 0_6 \\ -M_e^{-1}B_e \end{bmatrix} \]

\[ C = \nabla l|_{\theta_e} \begin{bmatrix} I_6 \\ 0_6 \end{bmatrix}. \]

3 CPG Architecture

With the dynamics of the structure complete, we can move onto the synthesis of a neural control system. Neural controls offer a robust solution to achieve a high level of autonomy in the control of robotic systems. Key characteristics that make artificial neural networks a prime candidate for control of an underwater vehicle are efficiency and adaptability. With the incorporation of sensory feedback, it should be possible for the control system to entrain to the structure’s resonance, allowing for large motions with minimal effort \(^{21}\). The CPG that will be used for this study is known as a reciprocal inhibition oscillator (RIO). It is a system of two neurons with mutually inhibitory synaptic connections. Each neuron, \( \mathcal{N} \), can be modeled as mapping from input \( w \) to output \( v \) as shown by \(^{21}\):

\[ q_i = F(s)w_i \]

\[ F(s) = \frac{2\omega_o s}{(s + \omega_o)^2} \]

\[ v_i = \varphi(q_i) \text{ for } i = 1, 2, \]

where \( \omega_o > 0 \). These are the dynamics of a band-pass filter, chosen to capture the lag and adaptation characteristic of neurons. The nonlinearity \( \varphi \) captures the saturation property of a neuron, and is assumed to be an odd, bounded, and strictly increasing function. It is strictly concave on \( x > 0 \) and \( \varphi'(0) = 1 \). A suitable choice for \( \varphi \) is the hyperbolic tangent function, \( \tanh(x) \).

We now connect two of these neurons together with inhibitory connections as shown in Fig. 4.

\[ q_i = F(s)w_i \]

\[ F(s) = \frac{2\omega_o s}{(s + \omega_o)^2} \]

\[ v_i = \varphi(q_i) \text{ for } i = 1, 2, \]

where \( \omega_o > 0 \). These are the dynamics of a band-pass filter, chosen to capture the lag and adaptation characteristic of neurons. The nonlinearity \( \varphi \) captures the saturation property of a neuron, and is assumed to be an odd, bounded, and strictly increasing function. It is strictly concave on \( x > 0 \) and \( \varphi'(0) = 1 \). A suitable choice for \( \varphi \) is the hyperbolic tangent function, \( \tanh(x) \).

We now connect two of these neurons together with inhibitory connections as shown in Fig. 4.

\[ q_i = F(s)w_i \]

\[ F(s) = \frac{2\omega_o s}{(s + \omega_o)^2} \]

\[ v_i = \varphi(q_i) \text{ for } i = 1, 2, \]

where \( \omega_o > 0 \). These are the dynamics of a band-pass filter, chosen to capture the lag and adaptation characteristic of neurons. The nonlinearity \( \varphi \) captures the saturation property of a neuron, and is assumed to be an odd, bounded, and strictly increasing function. It is strictly concave on \( x > 0 \) and \( \varphi'(0) = 1 \). A suitable choice for \( \varphi \) is the hyperbolic tangent function, \( \tanh(x) \).

We now connect two of these neurons together with inhibitory connections as shown in Fig. 4.

\[ q_i = F(s)w_i \]

\[ F(s) = \frac{2\omega_o s}{(s + \omega_o)^2} \]

\[ v_i = \varphi(q_i) \text{ for } i = 1, 2, \]

where \( \omega_o > 0 \). These are the dynamics of a band-pass filter, chosen to capture the lag and adaptation characteristic of neurons. The nonlinearity \( \varphi \) captures the saturation property of a neuron, and is assumed to be an odd, bounded, and strictly increasing function. It is strictly concave on \( x > 0 \) and \( \varphi'(0) = 1 \). A suitable choice for \( \varphi \) is the hyperbolic tangent function, \( \tanh(x) \).
4 Entrainment of RIO to Tensegrity

4.1 Oscillation Analysis using MHB

Consider the system shown in Fig. 5, where the transfer function between the \(i^{th}\) input (manufacturing length of string \(i\)) and the \(j^{th}\) output (current length of string \(j\)), be represented by \(P_{ji}(s)\). Integrating our RIO with these dynamics we develop the following equations:

\[
q_1 = F(s)(r_1 - \mu \varphi(q_2)),
q_2 = F(s)(r_2 - \mu \varphi(q_1)) \tag{7}
\]

\[
r_1 = hi_1, \ r_2 = hi_2 \tag{8}
\]

\[
\dot{i}_1 = P_{11}(s)\hat{i}_{o_1} + P_{14}(s)\hat{o}_{a_1}, \tag{9}
\]

\[
\hat{i}_4 = P_{14}(s)\hat{i}_{a_1} + P_{44}(s)\hat{a}_{o_4}, \tag{10}
\]

\[
\hat{o}_{a_1} = g\varphi(q_1), \ \hat{i}_{a_1} = g\varphi(q_2) \tag{11}
\]

![Figure 5: RIO-Tensegrity system](image)

We expect the signals in (7) to oscillate, so it will be useful to predict the frequency and amplitude of these oscillations. To this end, we use the Fourier series approximations of these signals, where a generic signal \(x(t)\) is represented by:

\[
x(t) = \sum_{k=0}^{\infty} [\alpha_k \sin(k\omega t) + \beta_k \cos(k\omega t)]
\]

We then take the phasor representation of the fundamental frequency component of the oscillation, denoted by \(\hat{x} = \alpha_1 + j\beta_1\). Applying describing functions to the nonlinearity \(\varphi(\cdot)\), we obtain an amplitude dependent gain approximation where

\[
\kappa_i := \hat{v}_i/\hat{q}_i. \tag{11}
\]

Here we must take a different approach than shown in section 5.1 of \(20\) because our two RIO output signals are not combined to form one output, nor are our feedback signals related by \(r_1 = -r_2\). In our system, each RIO output is used to dictate the manufacturing length of a different string in the structure. Therefore we cannot proceed with our analysis simply assuming that our signals \(q_i\) have the same time course with a possible phase shift because no significant insight can be gained. However, if we make a further assumption that \(q_1 = -q_2\), or that the two signals have a phase offset of 180°, we can proceed with the method of harmonic balance to gain insight on the system’s oscillations. This assumption is believed valid because simulations (Sec. 4.4) illustrate it to be true, as well as the fact that the RIO inherently oscillates 180° out of phase. Additionally, looking at the linearized dynamics of the tensegrity structure, we see that \(P_{11}(s) = P_{44}(s)\) and \(P_{14}(s) = P_{41}(s)\). This symmetry allows us to assume that if \(q_1 = -q_2\) the feedback to the RIO has the same time course with a phase shift of 180°. This assumption results in \(\kappa_1 = \kappa_2 := \kappa\). Using (11), (7) becomes

\[
\begin{bmatrix}
1 & \mu \kappa F(j\omega) \\
\mu \kappa F(j\omega) & 1
\end{bmatrix}
\begin{bmatrix}
q_1 \\
q_2
\end{bmatrix} = F(j\omega)
\begin{bmatrix}
r_1 \\
r_2
\end{bmatrix}. \tag{12}
\]

If \(q_1 = -q_2\), then \(\hat{i}_{a_1} = -\hat{i}_{o_1}\). It then follows that \(\hat{i}_1 = (P_{11}(s) - P_{14}(s))\hat{o}_{a_1}\), which in phasor representation is

\[
\hat{i}_1 = (P_{11}(j\omega) - P_{14}(j\omega))\hat{a}_{o_1}. \tag{13}
\]

Using (12) and \(\hat{i}_{o_1} = g\kappa \hat{q}_1\), we arrive at

\[
\hat{i}_{o_1} = g\kappa \frac{F(j\omega)}{1 - \mu \kappa F(j\kappa)} \hat{r}_1 = gh\kappa \frac{F(j\omega)}{1 - \mu \kappa F(j\kappa)} \hat{r}_1. \tag{14}
\]

Combining (13) and (14), we obtain

\[
1 = \kappa F(j\omega)(\mu + gh(P_{11}(j\omega) - P_{14}(j\omega))). \tag{15}
\]

Our \(F(j\omega)\) acts as a low pass filter, so it is reasonable to assume that higher order oscillation frequencies will be sufficiently attenuated. Keeping only the first harmonic of oscillation and recalling that our \(F(j\omega)\) blocks the DC component, we are left with

\[
\angle[F(j\omega)H(j\omega)] = 0, \tag{16}
\]
where \( H(j\omega) = \mu + gh(P_{11}(j\omega) - P_{14}(j\omega)) \). This shows that the closed loop system is expected to oscillate at a frequency that satisfies the phase angle condition (16).

### 4.2 Entrainment Predictions

The bode diagrams (Fig. 6) show entrainment is predicted for cases where \( \omega_0 \), the RIO’s intrinsic frequency, is both less than and greater than \( \omega_1 \), the first mode of the tensegrity structure. We see the desired behavior in \( H(j\omega) \), i.e. a sharp phase drop at resonance, and \( F(j\omega)^{-1} \) is relatively flat. Table 1 contains the predicted oscillation frequencies, \( \omega_p \). We expect robust entrainment no matter if \( \omega_1 \) or \( F(j\omega) \) shifts, for the phase intersection of \( H(j\omega) \) and \( F(j\omega)^{-1} \) will occur in roughly the same place.

#### Table 1: Simulated and predicted frequencies of oscillation (rad/sec)

<table>
<thead>
<tr>
<th>( \omega_0 )</th>
<th>( \omega_1 )</th>
<th>( \omega_p )</th>
<th>( \varepsilon_{ps} )</th>
<th>( \varepsilon_{ent} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>5.5886</td>
<td>5.4470</td>
<td>5.3729</td>
<td>1.36%</td>
</tr>
<tr>
<td>2.0</td>
<td>5.6771</td>
<td>5.7731</td>
<td>1.69%</td>
<td>1.58%</td>
</tr>
</tbody>
</table>

### 4.3 Robustness of Entrainment

To study the robustness of entrainment, we can consider perturbations in the prestress level, \( P \), of our structure (see 3). This can be conceptualized as an error in the manufacturing lengths, resulting in an error in the stiffness of the structure. Using the parameters \( \omega_0 = 12 \) and \( \eta = 2 \), we test whether the RIO is still able to entrain to the structure’s resonant mode. As shown by Fig. 7, the flatness of \( F(s)^{-1} \) results in an intersection between \( F(s)^{-1} \) and \( H(s) \) close to a phase angle of \(-90^\circ\), the location of mechanical resonance.

Table 2 presents the results from the robustness analysis, where

\[
\varepsilon_p = \frac{|\omega_p - \omega_1|}{\omega_1}.
\]

As shown, the error is relatively small, illustrating that entrainment of the control system is robust against perturbations in the pretension of the structure.

### 4.4 RIO Control Simulations

Simulations of the system shown in Fig. 8 are carried out using Matlab’s Simulink package. Note that the linearized tensegrity dynamics in this simulation
is different than that of Fig. 5, because Simulink can calculate the nonlinear relationship between $\Theta$ and $\tilde{l}$ at each time step.

![Simulated RIO controlled tensegrity system](image)

Figure 8: Simulated RIO controlled tensegrity system

Entrainment is achieved for both $\omega_o < \omega_1$ and $\omega_o > \omega_1$ as shown by the plots of $q_i$ in Fig. 9 and the plots of $\theta_1$ in Fig. 10. The simulated systems oscillated at frequencies $\omega_p$ shown in Table 1, and the results from the simulations are closer to the resonant frequency of the tensegrity than the predictions from the method of harmonic balance, where

$$e_{ps} = \frac{|\omega_p - \omega_s|}{\omega_s}, \text{ and } e_{ent} = \frac{|\omega_s - \omega_1|}{\omega_1}.$$

$$\text{(18)}$$

![Plots of neural activity from simulations](image)

Figure 9: Plots of neural activity from simulations

To illustrate the capabilities of the control system, we also checked to see if the RIO could entrain to the second mode, $\omega_2$, of the tensegrity system. As shown by the results plotted in Fig. 11 entrainment is verified, with the system oscillating at 40.8906 rad/sec, an error from $\omega_2$ of 3.06%.

![Entrainment to the second mode](image)

Figure 11: Entrainment to the second mode, $\omega_o = 20, \eta = -20$
5 Conclusion

We have shown that it is possible to control a tensegrity structure with a central pattern generator. First, a linear dynamic model of the tensegrity was generated and validated against the nonlinear model. Specific attention was paid to the modes of vibration as well as the behavior of the linearized model when controlled at resonance. Next, an RIO was simulated by joining two neuron models with inhibitory connections. Using the method of harmonic balance, robust entrainment was predicted for cases where the intrinsic frequency of the RIO was both less than and greater than the tensegrity’s first resonant mode. As discussed by Futakata et al.\textsuperscript{21} the mechanisms for robust entrainment can be viewed as positive rate feedback with saturation ($\omega_o > \omega_1$) and negative integral feedback with saturation ($\omega_o < \omega_1$). Robustness was then illustrated by considering variation in the level of prestress in the tensegrity. Next, numerical simulations were used to verify the predictions from the method of harmonic balance. Additionally, it was shown that the RIO was able to entrain to the structure’s second mode. Also important to note is the input output nature of the RIO-Tensegrity system. Our control is position, or length based (manufacturing length of cables), and our feedback is also position (current length). This is different than previous system models, where the input is a force (or torque) and the output is position. We are able to achieve entrainment because there is a dynamic, not static, relationship between the two signals (manufacturing and current length), and we exploit the resonance and corresponding phase shift for entrainment.

Appendix

Following the assumptions outlined above, the nonlinear inertia matrix is:

$$M(\theta) =\begin{bmatrix}
\frac{7}{3} & 0 & 0 & \frac{3}{2}c_{1,4} & \frac{2}{3}c_{1,5} & 0 \\
0 & \frac{7}{3} & \frac{2}{3}c_{2,3} & 0 & 0 & \frac{1}{2}c_{2,6} \\
0 & \frac{3}{2}c_{2,3} & \frac{3}{2} & 0 & 0 & \frac{1}{2}c_{3,6} \\
\frac{3}{2}c_{1,4} & 0 & 0 & \frac{4}{3} & \frac{1}{2}c_{4,5} & 0 \\
\frac{1}{2}c_{1,5} & 0 & 0 & \frac{1}{2}c_{4,5} & \frac{1}{3} & 0 \\
0 & \frac{1}{2}c_{2,6} & \frac{1}{2}c_{3,6} & 0 & 0 & \frac{1}{3}
\end{bmatrix},$$

where $c_{i,j} = l^2_m\cos(\theta_i - \theta_j)$.

Acknowledgements

The authors would like to acknowledge the support of VSGC, ARCS, NSF, and ONR.

References


34 N. Kanchanasaratool, D. Williamson, Motion control of a tensegrity platform, Communications in Information and Systems 2 (3) (2002) 299–324.


