STATE TRANSITIONS IN BLACK HOLE X-RAY BINARY DISKS

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ABSTRACT

In order to investigate the mechanism for state transitions in black hole X-ray binary disks, we have proposed a series of local shearing box simulations. These simulations will systematically examine the impact of various physical effects on the MRI-driven angular momentum transport in these disks. In particular, nonzero viscosity and resistivity and their ratio, the magnetic Prandtl number, have been shown to have a strong effect on the angular momentum transport. Previous studies have focused on a limited set of initial conditions. We expand on these initial conditions in this paper in order to gain a deeper understanding of exactly how viscosity and resistivity affect the accretion flow. We find that for a purely azimuthal magnetic field configuration, which is more applicable to real accretion disks, the angular momentum transport depends strongly on the resistivity, but is nearly independent of the viscosity. This work serves as an essential stepping stone to constructing simulations where the viscosity and resistivity are temperature-dependent and are thus controlled by turbulent heating, radiative cooling, and vertical gravity.

Introduction

Black holes are perhaps some of the most elusive objects in astrophysics, as nothing, not even light, can escape their mighty gravity. However, some black holes cannot remain hidden from our gaze. These black holes are known as black hole X-ray binaries (BHXRBs), which are systems in which the immense gravity of the black hole strips gas off of a companion star. The gas forms a disk around the black hole, loses angular momentum, and accretes inward towards the hole, releasing gravitational energy in the process. This gravitational energy is radiated in the form of X-rays, and it is this radiation that has made observations of accretion in BHXRBs possible with X-ray telescopes such as Chandra and XMM-Newton.

These observations have also revealed that BHXRBs cycle between two distinctive states. In one of the states, known as the hard state, these systems emit light in primarily high energy but low luminosity X-rays with a power law spectrum. In the second state, called the soft state, they emit light in low energy, high luminosity X-rays with a blackbody spectrum (e.g., Remillard & McClintock 2006)). In one model, the hard state is explained by a hot, low-density accretion flow near the black hole, which scatters low energy photons from a truncated cooler, high-density disk to higher energies. The soft state is explained by the cooler disk extending down to the black hole. This disk emits primarily as a blackbody, and there is no hot, low-density component to produce the power law spectrum (Done et al. 2007; Zdziarski & Gierliński 2004). See Fig. 1 for a visual depiction of this model.

It has long been suggested that the transition between these two states arises from a change in the gas accretion rate. For example, the hard state could result from a relatively high accretion rate that heats the gas enough for it to become hot and tenuous, whereas the soft state would originate from a smaller accretion rate and a cooler and denser disk. However, what could be causing such a change in accretion rate?

A long-standing problem in the study of accretion disks is understanding the mechanism that carries angular momentum away, allowing the gas to accrete. The last two decades have seen much progress in answering this question. It is now thought that turbulent stresses induced via the magnetorotational instability (MRI) is the primary mechanism for angular momentum transport (Balbus & Hawley 1991, 1998). This instability is extremely robust; all that is required for the excitation of turbulence is a relatively weak magnetic field coupled to gas with a negative outward angular
velocity gradient. Many numerical simulations have shown that angular momentum is transported outward at significant rates via this mechanism (e.g., Hawley et al. 1995, 1996; Stone et al. 1996; Hawley 2001).

Given the strong possibility of the MRI as the driving mechanism for accretion, it seems likely that the details of MRI-driven turbulence could play a role in state transitions. Many studies have been performed in order to quantify the dependence of the turbulence saturation level (and thus the accretion rate) on various physical parameters of the gas. These studies have used the so-called “shearing box” approximation, in which the simulation domain consists of a local corotating patch of accretion disk, small enough to be expand the MHD equations into Cartesian coordinates and ignore curvature terms (see Hawley et al. 1995). This approximation, in its simplest form focuses only on the basic physics of the MRI itself, which is useful in understanding what exactly sets the saturation level. Furthermore, global simulations (i.e., simulations of the entire disk) lack the numerical resolution necessary to resolve much of the small-scale physics.

Hawley et al. (1995) and Hawley et al. (1996) found that initial magnetic field geometry plays an important role, with a net vertical field through the domain producing the largest stress levels. Furthermore, the stress level also depends strongly on the strength of the magnetic field in the case of a vertical magnetic flux. There is also a dependence on the magnetic field strength for a net toroidal field configuration. Sano et al. (2004) used shearing box simulations to investigate the effect of gas pressure on the turbulence. They found an extremely weak dependence, the exact nature of which depends on the initial magnetic field geometry in the domain.

The processes that control the dissipation of turbulent energy into heat have been shown to have a profound impact on MRI driven turbulence. Simulations by Hawley et al. (1996), Sano et al. (1998), Fleming et al. (2000), Sano & Inutsuka (2001), Ziegler & Rüdiger (2001), and Sano & Stone (2002) have investigated the impact of a constant, nonzero Ohmic resistivity. The main result of these studies is that increasing the resistivity leads to a decrease in turbulence, independent of the initial field configuration. It has been recently shown that both the resistivity and the kinematic shear viscosity are important for setting the stress level in both models that preclude a net magnetic flux (Fromang et al. 2007) and vertical net field models (Lesur & Longaretti 2007). In particular, the saturation level increases with increasing magnetic Prandtl number, defined as $P_m = \nu/\eta$, where $\nu$ is the viscosity, and $\eta$ is the resistivity. This Prandtl number dependence itself depends on the Reynolds number of the flow. For example, Fromang et al. (2007) find that for the zero net field case, there exists a Prandtl number below which the turbulence dies out, and that this critical Prandtl number decreases with increasing Reynolds number (at least for the range of $\nu$ and $\eta$ values examined in the paper).

Such a dependence on the Prandtl number could be very important, as suggested by Balbus & Henri (2008) who put forth a model that explains the state transitions in terms of a “Prandtl number instability.” In this model, an increase in gas temperature due to turbulent heating causes a sharp increase in Prandtl number because of the steep dependence of the Prandtl number on the gas temperature, $P_m \propto T^4$. The increase in $P_m$ enhances the turbulence, which, through turbulent dissipation, then increases the temperature further, leading to a runaway. The end result is two separate regimes: one regime is hot and tenuous with $P_m \gg 1$, and the other is cooler and dense with $P_m \ll 1$. This entire process is also mediated by radiative cooling, as the temperature is determined by a balance between heating and cooling. Finally, the vertical component of the black hole’s gravity may play a significant role as it sets the temperature distribution.

![Fig. 1.— Likely geometry in the hard (a) and soft (b) X-ray states for an accretion disk around a black hole. In the hard state (a), the accretion flow near the black hole is dominated by a hot, low-density gas that scatters photons from a truncated cool, high-density disk to high energy X-rays. The soft state (b) consists of the cooler disk extending down to the black hole. This disk emits primarily as a blackbody. Note that this figure was taken from Zdziarski & Gierliński (2004) and includes some features not discussed in this paper (e.g., outflows and jets).](image-url)
Our aim in this project is twofold. First, we wish to further examine the Prandtl number dependence of the MRI turbulence saturation level. This dependence has barely been examined, but it is extremely important to understand in order to investigate the Prandtl number mechanism of Balbus & Henri (2008). In particular, a more thorough examination of the $P_m$ dependence for various magnetic field configurations should be carried out because the field configuration has a strong impact on the accretion rate, as described above. Furthermore, there are strong arguments for the dominant magnetic field in a given region of accretion disk being the toroidal field (see Guan et al. 2009), and this particular field configuration has not been examined in the context of a changing Prandtl number. Finally, we wish to probe the regimes, with amplitude $\delta \rho/\rho = 10^{-4}$. All simulations have resolution 128 grid points in the radial direction in our domain with $x$ corresponding to the center of the box. Similarly, $y$ and $z$ are the azimuthal and vertical directions respectively. In the present simulations, we assume a constant viscosity and resistivity as well as an isothermal gas. The evolution equations for the gas are given by:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot [\rho \mathbf{v} \mathbf{v} - \mathbf{BB} - \nu \nabla \mathbf{v}] + \nabla P + \frac{1}{2} \mathbf{B}^2 - \frac{1}{3} \nu (\nabla \cdot \mathbf{v}) = 2 \eta \Omega^2 x - 2 \Omega \times \mathbf{r},$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{v} \times \mathbf{B} + \eta \nabla \times \mathbf{B}) = 0,$$

where $\rho$ is the mass density, $\rho \mathbf{v}$ is the momentum density, $\mathbf{B}$ is the magnetic field, $P$ is the gas pressure, and $\nu$ and $\eta$ are the shear viscosity and Ohmic resistivity, respectively. Note that our system of units has the magnetic permeability $\mu = 1$. We use $q = 3/2$, appropriate for a Keplerian disk. The first source term on the right-hand side of equation (2) corresponds to tidal forces (gravity and centrifugal) in the corotating frame. The second source term in equation (2) is the Coriolis force. We have neglected the vertical gravity of the black hole for our current studies. Finally, our system of equations is closed by an isothermal equation of state, $P = \rho c_s^2$, where $c_s$ is the isothermal sound speed.

Our shearing box has a radial and vertical size $H$ and azimuthal size $4H$, where $H$ is the scale height of the disk. To account for the differential rotation of the gas in orbit, we initialize a velocity flow with $\mathbf{v} = -q \Omega x \mathbf{y}$, with $q = 3/2$, $\Omega = 0.001$, and $-H/2 \leq x \leq H/2$. The isothermal sound speed is $c_s = \Omega H$. Our equation of state gives us $P = \rho \Omega^2 H^2$, and we choose $\rho = 1$. Thus, we have $P = 10^{-6}$. In all simulations, we seed the MRI with random perturbations to $\rho$ with amplitude $\delta \rho/\rho = 0.01$. All simulations have resolution 128 grid points in the $x$ dimension, 200 points in the $y$ dimension, and 128 points in the $z$ dimension.

For comparison with previous results, we have run a series of simulations with zero net magnetic flux through the domain, initialized with $\mathbf{B} = \sqrt{2P/\beta} \sin[(2\pi/H) x] \mathbf{z}$ where $\beta = 400$. These runs are labelled SZ (for Sinusoidal Z-field). We vary the viscosity and resistivity in these simulations to reproduce the calculations of Fromang et al. (2007). The dissipation is quantified by the Reynolds number,

$$Re \equiv \frac{c_s H}{\nu},$$

magnetic Reynolds number,

$$Rm \equiv \frac{c_s H}{\eta},$$

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and the magnetic Prandtl number,

\[ P_m \equiv \frac{\nu}{\eta}. \]

(6)

We have also run a series of simulations initialized with a uniform toroidal field given by \( B = \sqrt{2P/\beta y} \), where \( \beta = 100 \). We again vary the viscosity and resistivity in these simulations to study the effect of dissipation coefficients on MRI turbulence.

**Results**

**Zero Net Flux Simulations**

For comparison with previous results (see Fromang et al. 2007), we have examined the effect of the Reynolds and Prandtl numbers on the MRI turbulence in the absence of a net magnetic field. Since these simulations were done with Athena whereas the simulations of Fromang et al. (2007) were performed with the ZEUS code, our results will serve to either support or call into question the previous results.

The results of these simulations show good agreement with those of Fromang et al. (2007). The values for \( \alpha \), which is the time- and volume-averaged stress divided by the gas pressure, agree well with the ZEUS-based results. The only major difference is that we find decaying turbulence for \( Re = 1600, P_m = 4 \), whereas ZEUS produces sustained turbulence for these parameters. The reason for this difference is unclear. However, it could be due to the fact that we use a slightly different domain size in \( y \) than used in Fromang et al. (2007). Despite this one case, we find good agreement with previous results, which supports those results as well as our implementation of explicit dissipation.

One of the main results of Fromang et al. (2007) was that viscosity (in addition to resistivity) plays an important role in determining the saturation amplitude. Indeed, it had been previously established that resistivity was important in determining the stress inside of a shearing box (Hawley et al. 1996; Fleming et al. 2000). We reiterate this viscosity dependence by plotting the time-averaged \( \alpha \) versus \( P_m \) at constant \( Rm = 12800 \) in Fig. 2.1 Time averaging was done from orbit 20 until the end of the simulation, and an orbit is defined at the radius of the center of our domain. The error bars denote the standard deviation about the time-averaged \( \alpha \). The increase in stress with viscosity is nearly linear, as observed by Fromang et al. (2007).

**Azimuthal Field Simulations**

We now turn our attention to the Prandtl number dependence for an initially pure azimuthal magnetic field.

\[ \text{Fig. 2.} \quad \text{Time-averaged \( \alpha \) parameter as a function of magnetic Prandtl number at constant \( Rm \approx 12800 \). The error bars denote one standard deviation about the time-average. Only the sustained turbulence simulations are plotted. There is a nearly linear increase in \( \alpha \) with viscosity.} \]

\[ \text{Fig. 3.} \quad \text{Volume-averaged magnetic energy density (normalized by the initial gas pressure) versus time for various \( P_m \) at a constant \( Re = 1600 \). The solid curve before } t = 36 \text{ orbits corresponds to a simulation with } Re = 25600 \text{ and } P_m = 4. \text{ We restarted from this simulation and changed the dissipation coefficients. The dot-dash line corresponds to } P_m = 4, \text{ the dashed line corresponds to } P_m = 2, \text{ the dotted line corresponds to } P_m = 1, \text{ and the solid line (after } t = 36 \text{ orbits) corresponds to } P_m = 0.5.} \]
configuration. Again, this magnetic field configuration is probably the most relevant to real accretion disks (see Guan et al. 2009), and therefore, it is important that we understand how the dissipation coefficients affect the angular momentum transport in this case.

In all of our azimuthal field simulations, we initialized the simulation with \( Re = 25600 \) and \( P_m = 4 \), and at orbit 36, we changed the values for \( Re \) and \( P_m \) to observe the affect of the changing dissipation values. Figure 3 shows the effect of changing the Prandtl number at a constant \( Re \) value (i.e., changing only the resistivity). The figure shows the volume-averaged magnetic energy density versus time. The magnetic energy density serves as a proxy for the turbulent stresses that drive accretion, because this turbulence pumps energy into the magnetic field. The resistivity has an obvious effect; increasing the resistivity causes a significant decrease in saturation level. For the highest resistivity used, the turbulence decays away completely.

Figure 4 shows the same Prandtl number affect but this time by keeping the \( Rm \) value constant. Thus, only the viscosity is changed here. In this case, changing the viscosity at a constant resistivity does not seem to have an impact on the saturation level. This is contrast to the case of zero magnetic flux, where the viscosity plays a significant role (i.e., Fig. 2).

**Conclusions and Future Work**

We have investigated the impact of the magnetic Prandtl number for both a zero net flux field configuration (for comparison with previous results) and an azimuthal field configuration. Our results indicate that the Prandtl number does indeed play a significant role. However, only the resistivity affects the angular momentum transport in the case of the azimuthal field.

Next, we plan to include a temperature dependence for the resistivity and viscosity, as well as vertical stratification, and radiative cooling in order to investigate the possible existence of a Prandtl number instability, as suggested by Balbus & Henri (2008). We also need to examine the \( P_m \gg 1 \) and \( P_m \ll 1 \) regimes.

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