Introduction
New spacecraft exploring the planets and moons of our solar system have greatly increased interest in atmospheric escape from planetary bodies. The Cassini spacecraft is currently improving our understanding of the atmospheres of the moons of Saturn. While in 2015 the New Horizons spacecraft will have a close flyby encounter with Pluto and the Maven mission will orbit Mars studying the composition of its escaping atmosphere. The motivation of this research is to produce an accurate model of Pluto’s atmosphere, which includes atmospheric loss by escape to be tested and validated against data obtained from this future encounter. By accurately describing present loss rates, one can hope to eventually be able to extrapolate back in time in order to describe the evolution of Pluto’s atmosphere. Doing this accurately for a planet for which we will have in situ spacecraft data will then guide our ability to model exoplanet atmospheres for which there will only be remote sensing data.

The previous models of atmospheric escape for Pluto used the concept called hydrodynamic escape by adapting the “critical” supersonic solutions that Parker (1964) describes. He was describing the expanding stellar corona and stellar wind, with temperature and pressure going to zero at infinity, and the upward fluid motion going supersonic. This model was adapted for planetary atmospheres and to include heating and developed into the theory of energy-limited escape and (Hunten & Watson, 1982).

More recently the hydrodynamic model has been applied to Pluto (Krasnopolsky 1999, Strobel 2008), Titan (Strobel 2009), and Triton. Being more strongly bound than stellar atmospheres, these atmospheres are more stable so that the model is often referred to as the slow-hydrodynamic escape (SHE) model. This SHE model and the energy-limited escape models both require solving the fluid equations to very large distances from the planet to enforce the boundary conditions. It is known that at some finite distance the equations of fluid fail to describe the flow of mass, momentum and energy in an atmosphere (Volkov, et al 2011).

The alternative to organized outflow is thermally driven escape, by which the atmosphere evaporates on a molecule-by-molecule basis driven by conductive heat flux from below. The standard analytic model of this mechanism was originally developed in Jeans (1925) and is referred to as Jeans escape. The escape rates are found to depend predominantly on the Jeans parameter $\lambda = GMm/rk_bT$, the ratio of gravitational to kinetic energy. The expected molecular escape and energy escape rates can be shown to be:

$$\frac{\phi}{2\sqrt{\pi}} = \frac{1}{2\sqrt{\pi}} \cdot 4\pi r^2 v_t(1+\lambda)e^{-\lambda} \cdot 4\pi r^2$$  \hspace{1cm} (1a)

$$\phi_{E_{esc}} = k_bT(2 + \frac{1}{1+\lambda})$$ \hspace{1cm} (1b)

Here $n$ and $T$ are the density and temperature, $v_t = \sqrt{2k_bT/m}$ is the most probable thermal speed and the Jeans parameter are all evaluated at the exobase (the altitude at which escape is most likely, defined later). From here on we drop the factor of $4\pi$ in the equation and understand the rates as per sterad.

Improved calculations of thermal escape can be obtained by using Direct-Simulated Monte-Carlo (DSMC) as in (Tucker & Johnson, 2009) and (Volkov, Johnson, Tucker, & Erwin, 2011). In this method, representative particles are used along with a model of molecular collisions with internal energy to calculate the temperature, density and other macroscopic values from the velocity distribution in both the dense and
rarified regions. In this respect, DSMC can model the exospheres with better accuracy than hydrodynamics model, but can only model the lower atmosphere at great computational cost.

With the arrival of the Cassini spacecraft to Saturn, many passes have been made of the moon Titan, which has a stable atmosphere of mainly nitrogen and methane. The average escape rate inferred from the data from many passes through the atmosphere disagreed by orders of magnitude between SHE and DSMC models of Titans upper atmosphere (Tucker & Johnson, 2009). Yet from these DSMC simulations and from the more thorough investigation in (Volkov, Johnson, Tucker, & Erwin, 2011), it was shown for atmospheres with Jeans parameter in excess of ~6 near the exobase the molecular escape rates does not deviate significantly from the Jeans rate.

Hybrid Simulations

Ideally one would like to obtain a DSMC solution Pluto’s entire atmosphere, modeling the transition from fluid regime to near-collisionless exosphere and how their interaction drives escape. From (Volkov, Johnson, Tucker, & Erwin, 2011) the escape rate can be determined in a reasonable amount of time for an atmosphere with Knudsen number of $Kn \sim 10^{-4}$ ($Kn = $ mean free path/atmospheric scale height) at the lower boundary. To model from a lower boundary deeper in the fluid regime, consistent with the lower boundary in the (Krasnopolsky 1999) and (Strobel 2008) models of Pluto, one would need to model down to $Kn \sim 10^{-7}$ or smaller. So currently it is computational impractical to rely on DSMC simulation alone. Neither can the fluid equations accurately model the entire atmosphere, as it does not accurately represent the tenuous, upper atmosphere.

Fluid-Jeans

At present there is a need for so-called hybrid solutions, which combine a fluid model of the lower atmosphere with a kinetic model of the upper atmosphere, with the transition below the exobase. A kinetic Monte Carlo model, like the DSMC model, will be the eventual choice for simulating the upper atmosphere, but (Volkov, et al 2011) has shown that the Jeans model is reasonably accurate in the range of Jeans parameters in Pluto and sized similar terrestrial-like objects. Using the Jeans model as an upper boundary condition we can investigate a large range of parameter spaces, and then refine some results with a fluid/DSMC model introduce later.

We use fluid equations for a single component, spherically symmetric atmosphere and ignore viscosity, as did (Parker, 1964), (Krasnopolsky, 1999) and (Strobel, 2008). As opposed to these authors, we use the time dependent form of the energy equation as (Zalucha 2011) and (Yelle 2003).

$$nC_p \frac{dT}{dt} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa(T) \frac{dT}{dr} \right) - \phi \left( C_p T + \frac{1}{2} m u^2 - \frac{G M m}{r} \right) - R_{NET}$$

Here $n$ is the number density, $u$ is the bulk velocity, $T$ is temperature, $\phi$ is the molecular escape rate, $m$ is mass of gas molecule, $k_b$ is Boltman’s constant, $\frac{G M m}{r}$ is gravitational potential energy, $C_p$ is specific heat ($\nu/k_b$ for diatomic), is conductivity, $R_{NET}$ is net radiative heating rate. In addition we use the steady state form of the mass and momentum equations

$$n u r^2 = \phi$$

$$\frac{dp}{dr} = p \frac{G M m}{r^2 k_b T}$$

with the pressure $p$ related through the equation of state $p = n k_b T$. As in Strobel (2008), I assume fixed mixing ratios of 97% N$_2$, 3% CH$_4$, and .0027% CO in thermal equilibrium. The net heating rate $R_{NET}$ is comprised of solar absorption by N$_2$ and CH$_4$ and rotational cooling of CO. RNET is
computed in the same method as (Strobel, 2008) using the UV and EUV flux values (Krasnopolsky, 1999) and the parameterized non-LTE heating and cooling given in (Strobel 2008).

For equation (2) we need 2 boundary conditions. At the lower boundary \( r = 1450 \text{km} \) we adopt a fixed temperature of 88.2K. At the top boundary we enforce a fixed energy flux to infinity, \( \phi_E \), by using

\[
r^2 \kappa(T) \frac{dT}{dr} \bigg|_{top} = \phi \left( C_p T - \frac{gMm}{r} \right) \bigg|_{top} - \phi_E
\]

The value of \( \phi \) and \( \phi_E \) we determined by using Jeans escape applied to the exobase. Since there is no heating between the exobase and the top boundary, the energy flux is constant so by enforcing energy flux at the top boundary we are also enforcing the energy flux at the exobase.

Using the substitution \( \xi = T^2 \), the energy equation (2) becomes linear in its derivatives of \( \xi \) and can be solved using finite differences. We modify the method of (Zalucha 2011) to include the intrinsic heat flow and to properly handle variable step sizes with finite differences, and obtain an implicit time stepping scheme for equation (2). After each time step equation (3a) and (3b) are used to calculate the density and bulk velocity from the updated temperature. The heating rate \( R_{\text{NET}} \) and fluxes \( \phi \) and \( \phi_E \) are updated using (1a) and (1b) the new temperatures and densities, and the next time-step is taken.

We use a radial grid equally spaced in \( 1/r \), so that it is more refined near the lower boundary and increases towards the top. Near the lower boundary the grid size is of the order 1km to resolve the sensitive temperature changes due to heating. Using larger grid spacing near the upper boundary saves significant computational cost. Several thousand time-steps (using \( dt = 10^{-5} \text{ sec} \)) are needed to get to steady state, but this only takes a few seconds on a personal computer. Solutions are found for many values of solar insolation to studying the expansion of the atmosphere due to heating.

Results from no heating to solar maximum conditions have been obtained, a region that includes the conditions that the New Horizon spacecraft will encounter in its 2015 encounter. We have shown that the resulting escape rate is consistent with the SHE model (ie Strobel 2008 and Krasnopolsky 1999) and can be approximated by energy-limited escape (McNutt 1989). The fluid/Jeans model produces a very different temperature and density structure from these previous models, and it more consistent with DSMC modeling of the exosphere. Therefore we conclude that our model is more representative of the atmosphere below the exobase than previous models. A paper is currently in preparation detailing this model and results, and will be submitted in the next couple months.

Fluid-DSMC

The hybrid model described above can be thought of as estimate or initial guess for coupled fluid and DSMC solutions. The DSMC model begins several scale heights below the exobase, typically near \( Kn=10^{-1} \), where the gas is still close to Maxwellian. The DSMC is run for a single component \( \text{N}_2 \) atmosphere and produces an estimate of escape flux and energy escape flux. For Pluto we use variable hard sphere (VHS) collision cross section to replicate the conductivity \( \kappa(T) = \kappa_0 T \), and Larson-Bornacke (LB) model internal energy to replicate the rotational modes of the diatomic \( \text{N}_2 \) (producing \( C_p=7/2k_b \) in equilibrium) (Bird 1994).

The DSMC model does not explicitly model the time evolution of the atmosphere, so it is not implemented simultaneously as the fluid. Instead the fluid/Jeans result is used as the initial profile, and a DSMC solution is run based off of this. The resulting \( \phi \) and \( \phi_E \) are then used to obtain a new fluid solution. This
iteration is performed until a consistent atmosphere is found; typically just a few iterations are needed. We qualify a consistent atmosphere when the escape rate and escape energy flux have settled to a few significant digits, as well as having a solution with a continuous $T$ and $dT/dr$ where the solutions are attached.

**Results**

First we present the results of the fluid-Jeans model for Pluto.

In figure 1 we show the temperature profile of the solution for the 4 special cases we investigate. Some relevant values of the solution can be found in the following table.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\phi$</th>
<th>$E_{\text{esc}}$</th>
<th>$r_{\text{exo}}$ (km)</th>
<th>$\lambda_{\text{exo}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.035</td>
<td>15.5</td>
<td>3909</td>
<td>8.7</td>
</tr>
<tr>
<td>min</td>
<td>1.13</td>
<td>16.9</td>
<td>6920</td>
<td>4.7</td>
</tr>
<tr>
<td>med</td>
<td>2.55</td>
<td>15.7</td>
<td>9535</td>
<td>3.7</td>
</tr>
<tr>
<td>max</td>
<td>5.77</td>
<td>13.2</td>
<td>15510</td>
<td>2.8</td>
</tr>
</tbody>
</table>

The units on $\phi$ are $10^{27}$ per sec, and the units on $E_{\text{esc}}$ are $10^{-3}$ eV. The total heat fluxes for these solutions are 0.0, 1.46, 2.99, and 6.00 x $10^{-3}$ ergs per cm$^2$ per sec.

We find that the escape rate for no heating is an order of magnitude lower than that found in pure fluid solutions (Strobel 2008). But in the presence on heating, our escape rates are typically ~10% smaller than the pure fluid solution. In the pure fluid solution, the typical assumption is that the energy carried off to infinity is zero; hence their $E_{\text{esc}}$ is zero.

The temperature structure for all cases is very different. This is due to the upper boundary conditions used. In a pure fluid solution, the temperature is forced to decrease fast to satisfy the zero temperature at infinity. We find higher exobase, as the density drops off less with altitude.

With the higher temperature and exobase, our Jeans parameter drops to below 10 at the exobase. This does not occur in the fluid models as easily since the Jean parameter is proportional to $r/T$ and at some altitude $T$ starts decreasing as $1/r$. Though having a Jeans parameter below 4 suggests that the solution is more in the fast outflow region defined in Volkov, et al (2011). The fluid equations are ignorant of this fact. As we will see later, the escape rate from DSMC is enhanced relative to Jeans in these cases so that the Jeans parameter does not drift into the hydrodynamic escape region.

From the fact that the escape rate is enhance relative to Jeans, we try to include by multiply the Jeans escape rate by 2. The following table shows the result of this enhancement.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\phi$</th>
<th>$E_{\text{esc}}$</th>
<th>$r_{\text{exo}}$ (km)</th>
<th>$\lambda_{\text{exo}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.053</td>
<td>15.3</td>
<td>3834</td>
<td>9.0</td>
</tr>
<tr>
<td>min</td>
<td>1.16</td>
<td>16.1</td>
<td>6124</td>
<td>5.5</td>
</tr>
<tr>
<td>med</td>
<td>2.55</td>
<td>14.9</td>
<td>8014</td>
<td>4.6</td>
</tr>
<tr>
<td>max</td>
<td>5.77</td>
<td>12.5</td>
<td>11998</td>
<td>3.7</td>
</tr>
</tbody>
</table>

The units are the same as in the previous table. We see that when there is significant heating that the escape rate does not change a by as much as a percent. The temperature structure
changes dramatically, resulting in the exobase being lowered by nearly 30%, and the Jeans parameter is held above the critical value of the hydrodynamic escape found in Volkov, et al (2011). The last two facts make this form of the solution more tractable. The solution at these higher heating rates being so sensitive can explain why different models have produced similar escape rates (Strobel 2008, Krasnopolsky 1999, Tucker 2012).

Fluid-DSMC

We move on to considering the fluid-DSMC hybrid simulation of Pluto. Here we replace the assumption of Jeans escape from an upper boundary, and instead use a DSMC simulation. The result is a valid solution well into the exosphere.

Figure 2: Temperature and density profile for the fluid-DSMC hybrid solution for the cases of no heating, solar minimum, and solar medium.

In Figure 2 we present the results of the fluid-DSMC. The solutions are only slightly different from the fluid-Jeans solution at the low altitudes. But below the exobase the DSMC solution begins to deviate from the fluid solution. This is where the molecules are becoming non-Maxwellian, the radial, orbital, and internal energies start to separate, and Fourier Law of heat conduction begins to fail (Volkov et al 2011, Tucker et al 2012). For these reasons the fluid equations should not be applied above the exobase. The following table has some relevant results for the Fluid-DSMC solution:

<table>
<thead>
<tr>
<th>Case</th>
<th>$\phi$</th>
<th>$E_{esc}$</th>
<th>$r_{exo}$ (km)</th>
<th>$\lambda_{exo}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>none</td>
<td>0.047</td>
<td>14.6</td>
<td>3875</td>
<td>8.9</td>
</tr>
<tr>
<td>min</td>
<td>1.20</td>
<td>14.3</td>
<td>6022</td>
<td>5.7</td>
</tr>
<tr>
<td>med</td>
<td>2.56</td>
<td>14.3</td>
<td>7716</td>
<td>4.8</td>
</tr>
</tbody>
</table>

When comparing these results to previous ones, we notice that the escape rate and energy being carried off does not change significantly. But these changes in the escape parameters are enough the keep the exobase low, and the Jeans parameter well above the region of hydrodynamic escape.

The ratio of escape rate to the theoretical Jeans value is 1.6, 2.0 and 2.3 for the three cases considered. We see that as we increase the heating rate we increase the enhancement relative to Jeans. In the fluid-Jeans model we consider escape occurring from a single altitude, the exobase. While the DSMC model shows that the source of the escaping molecule is distributed through the whole atmosphere, but is focused around the exobase.

In figure 2 we can see that a gradual cooling continues well into the exosphere, but not as large as found in a pure fluid solution. Conduction in the upper atmosphere is small as there occurs few collisions. This cooling can be thought of as near isentropic expansion of the atmosphere.

Conclusions

We defined two hybrid models for studying the upper thermosphere of an atmosphere. We found that using Jeans escape from the upper boundary approximates the more complex DSMC model of exosphere. In particular the escape rate and temperature structure are close, but some important values such as the exobase are not as well constrained.
Both of these models produce a more extended atmosphere than previous models. The New Horizons spacecraft should be arriving at a time when the solar flux is between the solar minimum and medium cases considered in this study. As its closest approach will be roughly 10000 km, it will still be outside the exobase. But during its encounter it will have a good chance to study the atmosphere via an occultation between the sun and Pluto.

Further models should consider multiple species as one would expect methane to diffuse through nitrogen and preferentially escape since it is lighter. We plan to carry out fluid-DMSC model of this, and study if a fluid-Jeans method can achieve a similar result.

References