AERODYNAMIC AND AEROELASTIC OPTIMIZATION OF HARMONICALLY DEFORMING THIN AIRFOILS FOR MAV DESIGN

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Abstract

The present paper addresses the aeroelastic optimization of a membrane wing. Aerodynamic optimization of a deforming wing has been done in recent work by Walker, Patil, and Canfield. The deformation shapes required for maximum thrust and thrust efficiency were calculated. In the present work, the maximum thrust for the membrane wing is calculated by optimizing the tension in the membrane so that the aeroelastic deformation due to wing motion will lead to optimal thrust and/or efficiency. A function which describes the variation of spanwise tension along the chord is calculated. It is shown that one can always find a range of membrane tension for which the flexible membrane wings performs better than the rigid wing. These results can be used in preliminary flapping wing MAV design.

Introduction

Much research has been done in the design of flapping wing Micro Air Vehicles (MAVs). Flapping wing MAV wings have high frequency oscillations and significant membrane deflections. Designing these type of flapping wing aircraft capable of producing thrust requires an aeroelastic theory which models the forces and kinematics of flapping membrane wings. Flapping wing MAVs generate lift and propulsion from flapping similar to bird flight. A feasible design can be obtained only if we design these flapping membrane wings such that they generate maximum thrust and efficiency. The mass and stiffness properties of the wing itself can be chosen such that flapping yields this optimum condition. Tailoring the tension in the membrane is one such way of achieving this goal. Aeroelastic tailoring of MAV wings is based on optimization of an aeroelastic system composed of an unsteady aerodynamic theory coupled with a membrane structural analysis.

The aerodynamics of deformable wings has been investigated previously by a variety of researchers. An unsteady deformable thin airfoil theory was developed by Walker and Patil based on the work of Theodorsen and Garrick. The pressure difference across a thin airfoil undergoing harmonic rigid body motion and harmonic deflections was derived along with the corresponding lift, pitching moment and thrust. The stroke-averaged thrust was calculated and presented in a quadratic matrix form. A similar theory developed by Peters presented the same results which were obtained via a different approach.

In order to generate suitable thrust from flapping, the proper combination of magnitude and phase of each deformation shape is required. Walker and Patil showed that the magnitude of each shape and the phase angles between them have a very large contribution to the value of thrust. Similar results were shown by Garrick for the case of a rigid airfoil undergoing pitching and plunging only. These results were validated in water tunnel experiments by Anderson et al. Heathcote and Gursul later found via water tunnel experiments that a peak in thrust occurred at a particular value of phase angle between pitch and plunge.

Optimizing the aerodynamics yields the magnitude and phase of each motion type (pitch, plunge, and deformations) required for a deformable wing to generate maximum thrust and efficiency. Tuncer and Kaya optimized a rigid flapping wing with a NACA0012 airfoil section for maximum thrust and propulsive efficiency using a Reynolds average Navier-Stokes (RANS) aerodynamic model. The design variables included magnitude of pitch and plunge motions and the phase angle between them.

Optimization of the trajectory of motion and the airfoil shape was conducted by Lee and Liou. The motion and shape were parameterized and the optimal parameters were found which yielded the maximum thrust. The optimization showed that there exists a periodic, but not sinusoidal trajectory which gives optimum thrust, which is larger than the thrust generated by the sinusoidal trajectory. Follow up work by Lee and Liou used a different parameterization scheme which better represented non-harmonic trajectories observed in nature.

Thrust and efficiency optimization of a harmonically deforming thin airfoil with motion constraints was presented by Walker, Patil and
The work described on aeroelastic analysis of membrane wings has shown that the characteristics of the structure can significantly change the resulting deflections and thus the effective aerodynamic forcing. Optimization of an aeroelastic membrane wing is thus critical for flapping wing MAV design. Aeroelastic optimization has been investigated previously. Kaya, Tuncer, et al.\textsuperscript{4} conducted numerical gradient based optimization of an aeroelastic flapping wing in a biplane configuration. Conditions for maximum thrust with RANS aerodynamics were presented using pitch and plunge amplitude and the phase angle between them as design variables. Further work by Kaya, Tuncer, et al.\textsuperscript{5} using the same optimization techniques and aerodynamics model found conditions for maximum thrust and propulsive efficiency. Their results were compared with a single flapping airfoil and it was shown that maximum thrust is larger for a biplane configuration.

A pseudo-analytical approach to the aeroelastic optimization problem has been conducted by Stanford and Beran\textsuperscript{16}. Their work investigated unsteady aeroelastic phenomena of flow around a nonlinear shell. Multiple optimization examples were presented including maximizing the thrust due to plunging, minimizing the amplitude of Limit Cycle Oscillation (LCO) and optimizing for passive gust load alleviation. The design space consisted of variables describing the thickness distribution and the mass was constrained to be the same value or less than the unoptimized wing.

Similar work in optimization with the use of analytic sensitivities was conducted by Maute, Nikbay and Farhat\textsuperscript{11}. They present an analytic sensitivity analysis of the aeroelastic problem and applied it to the optimization of three dimensional wings.

Lian, Shyy and Haftka\textsuperscript{10} presented a shape optimization of a membrane wing for MAVs. The optimization maximized the lift to drag ratio of a rigid fixed wing MAV with battons. The design variables consisted of parameters representing the camber of the wing. While the optimization was done for a rigid wing MAV, it was shown that optimization of a rigid wing shape can improve the performance of a flexible wing with the same geometry.

Purely numerical approaches to aeroelastic optimization have been conducted. Nikbay, Öncü and Aysan present a methodology and code for aeroelastic optimization with high-fidelity commercially available software\textsuperscript{13}. Most of the previous research into the analysis and optimization of aeroelastic wings has been done using computational analyses. These efforts are very useful and the results generated have shown that the...
design of the structure has a significant effect on the deformation and forces generated for an aeroelastic membrane wing. However, these techniques are also expensive and time consuming. Furthermore, most of the optimization research has been applied to beam/shell fixed wings.

The current paper presents the complete aeroelastic system of a flapping membrane wing including unsteady aerodynamic forcing in a nondimensional form. The complete optimization problem is posed where thrust is being optimized and the design space consists of the tensions in the membrane. Aeroelastic optimization is conducted on a single degree of freedom (quadratic camber) and a three degree of freedom system (quadratic, cubic and quartic camber). The single degree of freedom system is used primarily to show the physical nature of the problem and the effect of changing frequency and mass of the wing relative to the surrounding fluid. The three degree of freedom system is used to show how allowing the tension to vary along the chord leads to a larger thrust than a constant tension.

Aerodynamic Model

The aerodynamic model is based on two dimensional unsteady potential flow theory which was previously developed in Walker and Patil\(^{19}\). The theory accounts for rigid body motion (pitch and plunge) as well as deformation. Consider a harmonically deforming thin deformable airfoil such as Figure 1. The deformation shapes are represented by a linear combination of basis functions (or assumed modes). Chebychev Polynomials are used as the basis functions in the present study and the general shape of the airfoil is given by,

\[
w(x,t) = \sum_{i=0}^{N-1} T_i(x)h_i(t) \tag{1}\]

where \( x \) is the nondimensional chordwise coordinate and \( w \) is the airfoil motion/deformation. \( h_i \)'s represent the generalized coordinates corresponding to each modeshape. The assumed modes, \( T_i \)'s, are the Chebychev polynomials which are given as,

\[
\begin{align*}
T_0(x) &= 1 \\
T_1(x) &= x \\
T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x) \tag{2}
\end{align*}
\]

The frequency domain unsteady deformable thin airfoil theory\(^{19}\) gives the stroke-averaged thrust on the airfoil as,

\[
T_{\text{average}} = \frac{1}{2} \rho u^2 b \Re \left[ \bar{h}^T \left[ TM \right] \bar{h} \right] \tag{3}
\]

where the matrix \([ TM ]\) is a function of reduced frequency and the vector \( \bar{h} \) are complex quantities representing magnitude and phase of each motion. The components of \([ TM ]\) are given in Walker and Patil\(^{19}\).

Aeroelastic Model of a Membrane Wing

Consider a membrane wing in an airflow as shown in Figure 2. The membrane can be prestressed in spanwise, chordwise and shear directions. The deflection, \( W(X,Y,t) \), is the deformation which includes rigid body motion (pitch and plunge) along with the membrane deformation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{membrane_wing.png}
\caption{Membrane Wing in Air}
\end{figure}

The aerodynamic forcing, \( \Delta P(X,Y,t) \), is effective load due to prescribed rigid body motion as well as membrane deformation. The equation of motion for this aeroelastic system is given as,
The aerodynamic problem of Eq. 6 can be reduced to a much simpler form by assuming the form of the vibration response in the spanwise direction. By making such an assumption, the spanwise variable, \( Y \), can be completely eliminated from the membrane problem. This will reduce the three dimensional membrane aeroelasticity problem to a two dimensional problem similar to string vibration.

Consider a wing with prescribed plunging and pitching. Let us discretize the membrane deformation in spanwise direction using sinusoids. Furthermore, let us assume that only one spanwise shape is sufficient to represent the the spanwise deformation variation. The assumed wing motion including rigid body motion can be represented in the following form.

\[
W(X,Y,t) = A_0(t) + A_1(t)X + W_D(X,t)\sin\left(\frac{\pi Y}{s}\right)
\]

The aerodynamics from strip theory using this assumed modeshape will result in

\[
\Delta P(X,Y,t) = \Delta P_R(X,t) + \Delta P_D(X,t)\sin\left(\frac{\pi Y}{s}\right)
\]

where \( \Delta P_D \) and \( \Delta P_R \) are the pressure due to airfoil deformation and rigid body motion respectively.

Substituting the assumed solution and aerodynamics into the membrane vibration problem results in

\[
S_y \sin\left(\frac{\pi Y}{s}\right)\frac{\partial^2 W_D(X,t)}{\partial X^2} - \\
\frac{\pi^2}{s^2} S_y (X) \sin\left(\frac{\pi Y}{s}\right) W_D(X,t) + \\
\Delta P_D(X,t) \sin\left(\frac{\pi Y}{s}\right) + \Delta P_R(X,t) = \\
\rho_w t_w \left[ \frac{\partial^2 A_0(t)}{\partial t^2} + \frac{\partial^2 A_1(t)}{\partial t^2} X + \frac{\partial^2 W_D(X,t)}{\partial t^2} \sin\left(\frac{\pi Y}{s}\right) \right]
\]

The spanwise variable can be eliminated using the weighted residual method. Using the assumed spanwise modeshape as the weight and integrating over the span results in

\[
\int \left[ S_y \sin\left(\frac{\pi Y}{s}\right) \frac{\partial^2 W_D(X,t)}{\partial X^2} - \\
\frac{\pi^2}{s^2} S_y (X) \sin\left(\frac{\pi Y}{s}\right) W_D(X,t) + \\
\Delta P_D(X,t) \sin\left(\frac{\pi Y}{s}\right) + \Delta P_R(X,t) \right] \sin\left(\frac{\pi Y}{s}\right) dY = \\
\int \left[ \rho_w t_w \left( \frac{\partial^2 A_0(t)}{\partial t^2} + \frac{\partial^2 A_1(t)}{\partial t^2} X + \frac{\partial^2 W_D(X,t)}{\partial t^2} \sin\left(\frac{\pi Y}{s}\right) \right) \right] \sin\left(\frac{\pi Y}{s}\right) dY
\]

The resulting equation looks exactly like the
string vibration equation with one extra term.

\[ S \frac{\partial^2 W_d(X,t)}{\partial x^2} - \frac{\pi^2}{s^2} S \phi(X) W_d(X,t) + \Delta P_\phi(X,t) + \frac{4}{\pi} \Delta P_R(X,t) = \rho_w u^2 \frac{\partial^2 W_d(X,t)}{\partial t^2} \]  \hspace{1cm} (11)

The reduced aeroelastic membrane equation can be further simplified by nondimensionalizing. Making substitutions of \( X = x b \), \( W_d = \omega_t A_0 \), \( A_t = a_t \), \( \tau = \omega t \), \( \Delta P(x,k) = \frac{1}{2} \rho_w u^2 \Delta p(x,k) \) and dividing by \( \rho_w u^2 \omega_t^2 \), the equation becomes

\[ \frac{S}{\rho_w u^2 \omega_t^2} \frac{\partial^2 w(x,\tau)}{\partial x^2} - \frac{S_1(x)}{\rho_w u^2 \omega_t^2} \pi^2 w(x,\tau) + \frac{1/2 \rho_w u^2 \Delta P_d(x,\tau) + 4/\pi \rho_w u^2 \Delta P_R(x,\tau)}{\rho_w u^2 \omega_t^2} = \frac{\partial^2 a_0(\tau)}{\partial t^2} + \frac{\partial^2 a_1(\tau)}{\partial t^2} \]  \hspace{1cm} (12)

Now assume that the prescribed motion is harmonic with the reference frequency, \( \omega \). Since the problem is linear, the solution can be written as,

\[ w(x,\tau) = w(x) e^{i\tau} \]

\[ a_0(\tau) = a_0 e^{i\tau} = \bar{a}_0 e^{i\tau} \]

\[ a_1(\tau) = a_1 e^{i\tau} = \bar{a}_1 e^{i\tau} \]

(13)

(14)

Thus, the frequency-domain, nondimensional, aeroelastic membrane equation is

\[ \bar{w} + q_x \bar{w} - \pi^2 q_x(x) \bar{w} + \frac{1}{\sigma k^2} \Delta \bar{P}_d(x,k,\bar{w}) = -\frac{4}{\pi} \frac{1}{\sigma k^2} \Delta \bar{P}_r(x,k,\bar{a}_0,\bar{a}_1) + \bar{a}_0 + \bar{a}_1 \]  \hspace{1cm} (15)

The nondimensional stiffness terms are

\[ q_x = \frac{S}{\rho_w u^2 \omega_t^2} \]

\[ q_y = \frac{S}{\rho_w u^2 \omega_t^2} \]  \hspace{1cm} (16)

and \( \sigma \) is the mass ratio.

\[ \sigma = \frac{\rho_d \lambda_x}{1/2 \rho_w b} \]  \hspace{1cm} (17)

The most convenient solution technique is the assumed modes method. The assumed modes solution is

\[ w(x,\tau) = \sum_{i=2}^{N+1} z_i(x) a_i(\tau) \]  \hspace{1cm} (18)

where \( z_i(x) \) are the assumed modeshapes, \( a_i(\tau) \) are the magnitudes of each modeshape and \( N \) is the total number of assumed modeshapes including the two rigid modes. The membrane vibration problem now becomes

\[ \sum_{i=2}^{N+1} z_i(x) \bar{a}_i + q_x \sum_{i=2}^{N+1} \pi^2 Q_i(x) \bar{a}_i - \pi^2 q_x(x) z_i(x) \bar{a}_i + \frac{1}{\sigma k^2} \Delta \bar{P}_d(x,k) + \bar{a}_0 + \bar{a}_1 \]  \hspace{1cm} (19)

Using the Galerkin method the continuous system can be reduced to a series of coupled linear equations for \( N-2 \) assumed modeshapes. Using the assumed modeshape as the weight and choosing a functional form of the normalized spanwise tension, \( q_y(x) \), the equation can be rewritten as

\[ \left[ M \right] - \left[ K \right] q_y(x) + \frac{1}{\sigma k^2} \left[ P_d(k) \right] \bar{a}_i = \frac{1}{\pi} \left( \frac{1}{\sigma k^2} \Delta \bar{P}_r(x,k) + \bar{a}_0 + \bar{a}_1 \right) \]  \hspace{1cm} (20)

where the mass, stiffness and inertial forces are

\[ \left[ M \right] = \int z_i(x) z_j(x) dx \]

\[ \left[ K \right] = \int q_x(z_k(x) z_j(x) - q_y(x)) \frac{\partial^2 z_i(x)}{\partial x^2} \]  \hspace{1cm} (21)

\[ \left[ F_i \right] = \int z_i(x) z_k(x) dx \]

where the indices range are \( 2 \leq k \leq N-1 \), \( 2 \leq l \leq N-1 \) and \( 0 \leq m \leq 1 \). \( \left[ P_d \right] \) and \( \left[ P_r \right] \) give the generalized forces corresponding to the structural generalized coordinates due to deformation and rigid body motion respectively.

The pressure difference was derived in the frequency domain using the Chebychev polynomials. However, these polynomials do not satisfy the boundary conditions given in Eq. 5. For structural representation we use orthogonal polynomials which satisfy boundary conditions. The first three orthogonal polynomials that satisfy the boundary conditions are:

\[ z_2 = x^3 - 1 \]

\[ z_3 = x^3 - x \]

\[ z_4 = x^3 - \frac{8}{7} x^2 + \frac{1}{7} \]  \hspace{1cm} (22)

Since the aerodynamic theory uses Chebychev polynomials, the pressure difference can be written as a function of corresponding \( h_i \)'s, and reduced frequency, \( k \).
\[ \Delta \rho(x,k) = \sum_{j=0}^{N-1} \Delta p_j(x,k) h_j \]  
(23)

Therefore the generalized forces for motion described by Chebychev polynomials are

\[ \vec{P}_k = \sum_{j=0}^{N-1} \int \Delta p_j(x,k) h_j T_j(x) dx = [P^k(h)] h_j \]  
(24)

where \([P^k(h)]\) is a function of only reduced frequency.

Eq. 24 gives the generalized forces for generalized coordinates based on the Chebychev polynomials. The structural equations require the generalized forces in terms of the structural assumed modes. Therefore a transformation matrix is needed. The transformation between the two generalized coordinates can be written as

\[ h_j = [C_{ji}] a_i \]  
(25)

where,

\[
[C_{ji}]=
\begin{bmatrix}
1 & 0 & -\frac{1}{2} & 0 & -\frac{3}{36} \\
0 & 1 & 0 & -\frac{1}{4} & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{8}
\end{bmatrix}
\]  
(26)

Transforming the generalized force matrix

\[ [P^H^j] = [C_{ji}] [P^h] [C_{ij}] \]  
(27)

The aerodynamic matrices required for the analysis above are:

\[ P_D = P_{3-N,3-N} \]
\[ P_R = P_{3-N,1-2} \]  
(28)

Analysis and Optimization of a 1 Degree of Freedom Membrane Wing

The system of equations in Eq. 20 provide the basis for optimization of the aeroelastic system for any number of assumed modes and any functional form of the spanwise tension. However, much can be learned from a simpler 1 degree of freedom model. Assuming a rigid body motion consisting of only plunge and the membrane vibrating in the first assumed mode, \(a_1\), the aeroelastic system reduces to a single equation for the complex magnitude of the deformation given by,

\[ \bar{a}_z = -\frac{4}{\pi} \frac{1}{M - K + \frac{1}{\sigma k^2} P_D} \bar{P}_D \]  
(29)

where the stiffness, \(K\), contains tension terms, \(q_s\) and \(q_y(x)\), \(M\) is the normalized mass term and \(F_I\) is the nodal force term. The aerodynamic term, \(P_D\) is the nodal force due to \(\bar{a}_z\) in the \(\bar{a}_z\) direction and \(P_R\) is the nodal force due to \(\bar{a}_0\) in the \(\bar{a}_0\) direction.

Eq. 29 gives the complex magnitude of the deformation for a prescribed plunging motion at a specific reduced frequency and mass ratio. Using the plunge and deformation information in Eq. 25 will give the complex magnitude of the motion in terms of the aerodynamic deformation shapes. This information can be used in Eq. 3 to obtain the average thrust for the aeroelastic system. Therefore specifying a rigid body motion, reduced frequency and mass ratio pose s the optimization problem where stiffness is the design variable.

Consider a case where the spanwise tension in the membrane is also constant, \(S_y(X) = S_{y0}\) or \(q_y(x) = q_{y0}\).

For reduced frequency values of zero \(K - 1/\sigma k^2 P_D\) is the aeroelastic stiffness. However, the stiffness is nondimensionalized using frequency, Eq. 16. Therefore we can rewrite the nondimensional stiffness in terms of different variables.

\[ K = \frac{1}{\sigma k^2} \left( \frac{8}{3} \tilde{q}_s + \frac{16}{15} \tilde{q}_{y0} \right) \]  
(30)

where,

\[ \tilde{q}_s = \frac{1}{2} \rho_s u^2 b \]
\[ \tilde{q}_{y0} = \frac{1}{2} \rho_s u^2 s^2 \]  
(31)

The aeroelastic stiffness is now

\[ -K + \frac{1}{\sigma k^2} P_D = \frac{1}{\sigma k^2} \left( \frac{8}{3} \tilde{q}_s + \frac{16}{15} \tilde{q}_{y0} - P_D \right) \]  
(32)

Ensuring no aeroelastic divergence is a good constraint on the problem, thus the aeroelastic stiffness is constrained to be positive. The membrane tensions required to generate the maximum possible thrust without having aeroelastic divergence can now be calculated.
Figures 3 and 4 show the effect of stiffness on the stroke-averaged thrust for a deformable wing in unit harmonic plunging. The added grid line above zero in Figure 3 represents 1. As expected, the stiffness has a significant influence on the thrust generated by the membrane wing. Typically the thrust generated by the flexible membrane wing has large changes below the divergence stiffness. Above the divergence stiffness though the thrust typically increase starting from negative values to peak at values in excess of the rigid wing thrust. Then as the stiffness further increases, the thrust generated asymptotically approaches the rigid wing results. For all cases, one can choose a range of stiffness for which the flexible wing outperforms the rigid wing. Figure 3 shows that for a given mass ratio the results change significantly with reduced frequencies. Figure 4 shows that for a given reduced frequency, the results do not change as much with mass ratio. The stiffness is comprised of parts due to the chordwise tension and spanwise tension as shown in Eq. 30. Stiffness is a scalar for the 1 degree of freedom system, and so performing a multi variable optimization is redundant. However, this does show that any combination of spanwise and chordwise tensions in the form of Eq. 30 that produce the stiffness equal to the value required for maximum thrust is a viable design solution.

The maximum thrust as a function of reduced frequency for various values of mass ratios is given in Figure 5. The corresponding stiffnesses are plotted in Figure 5. The optimization routine, fmincon in MATLAB was used to generate the results. It can be seen that a flexible wing can always be designed to yield a larger thrust as compared to a rigid wing in a unit harmonic plunging motion. For a given mass ratio, there is a specific reduced frequency at which the increase in thrust is a minimum with the increase being high for both low and high reduced frequencies. Increasing frequency for a specific mass will make the added mass component of the aerodynamics larger which leads to a larger deformation and therefore a larger thrust. However, decreasing the frequency leads to the maximum thrust occurring at a lower stiffness because
the wing is more compliant. As reduced frequency approaches zero, the stiffness for maximum thrust approaches the stiffness which will lead to divergence, which leads to the larger increase of thrust over the rigid body case.

Increasing the mass ratio also changes the maximum average thrust. A heavier wing will lead to more thrust than a lighter one when a unit plunge is prescribed. This is due to the inertial effects of the wing itself. A heavier wing will deform more, increasing the magnitude of deformation, which will then increase the maximum thrust.

The one degree of freedom case described above provides a very useful approximation for tensions required in a membrane wing such that maximum thrust can be generated when flapping. However, the actual wing will deform in higher modes as well and the thrust can be accurately captured only if more degrees of freedom are considered. Furthermore, more degrees of freedom leads to a stiffness matrix with multiple stiffness numbers which can be tailored. This is achieved by having a nonuniform spanwise tension.

Analysis and Optimization of a 3 Degree of Freedom Membrane Wing

The one degree of freedom results show much insight into the physics of the aeroelastic system. However, realistic systems will vibrate in more than one degree of freedom. Consider a three degree of freedom system. The mass, stiffness and forcing matrices are determined from Eq. 21. The deformations can be written

\[
\begin{bmatrix}
\pi_1 \\
\pi_2 \\
\pi_3
\end{bmatrix}
= \frac{4}{\pi} \left( [M]-[K] + \frac{1}{\sigma k^2} [P_0] \right)^{-1} \left( \frac{1}{\sigma k^2} [P_1] + [F] \right) \begin{bmatrix}
\pi_1 \\
\pi_2 \\
\pi_3
\end{bmatrix}
\]  (33)

The stiffness matrix is a function of both the chordwise tension, \( q_x \), and the spanwise tension, \( q_y(x) \).

As with the one degree of freedom system, a constraint can be placed on the system such that divergence never occurs. In a multi degree of freedom problem, divergence calculation is an eigenvalue problem. Ensuring that divergence never occurs requires that the aeroelastic stiffness to always be positive definite. The aeroelastic stiffness can be expressed in terms of the alternate nondimensionalized variables, \( \tilde{q} \) just like the one degree of freedom system.

\[
[K(\tilde{q})] - \frac{1}{\sigma k^2} [P_D] = \frac{1}{\sigma k^2} ([K(\tilde{q})] - [P_D]) > 0
\]  (34)

There will be three divergence modes, however only the mode which diverges at lowest velocity or highest stiffness is critical. The stiffness at which the smallest eigenvalue of the matrix \( ([K(\tilde{q})] - [P_D]) \) becomes negative is the stiffness where divergence will occur.

Increasing the number of degrees of freedom of the system does not change the system such that it can be tailored for more thrust. It simply models the physics of the system more accurately. The only way to achieve larger thrust is by expanding the design space by having the spanwise tension take a more general form.

Let us represent the nondimensional tension, \( q_y(x) \), is terms of a polynomial expansion

\[
q_y(x) = \sum_{i=0}^{M} q_{y_i} x^i
\]  (35)

The optimization problem now has \( N + 2 \) design variables. The stiffness matrix is a function of these design variables. However, for the three degree of freedom system the stiffness matrix can only have 6 independent terms due to symmetry. Therefore having more than 6 design variables is redundant. The largest feasible design space for the three degree of freedom system is \( q_x, q_{y_0}, q_{y_1}, q_{y_2}, q_{y_3}, \) and \( q_{y_4} \).

Consider the two design variable case where the spanwise and chordwise tensions are constant, but are not required to be equal to each other. During studies of the two variable problem it was found that multiple maxima existed and the optima calculated was dependent on the initial guess provided. Figure 7 shows the improvement in optimal thrust by expanding the design space from one to two variables. The results presented use the optimal but equal tensions from the one design variable case as the starting point for the optimization. This always leads to improvement in thrust performance. However, there is no guarantee that the solution is the global maximum. The two design variable system shows improvement over the one design variable
system for both a small and large mass ratios over a range of reduced frequency. Therefore allowing the membrane wing to have different tensions in the spanwise and chordwise directions allows for the structure to be tailored so that more thrust can be produced during pure plunging.

Finally, consider a third case with three design variables, $q_x$, $q_y^0$, and $q_y^1$. The design space for the three variable system allows for a linear variation in the spanwise tension. Figures 8 and 9 show the optimized stroke-averaged thrust and corresponding nondimensional spanwise tension at various reduced frequencies.

Expanding the design space to have linear spanwise tension variation along the chord significantly increases the amount of thrust possible for an airfoil undergoing unit plunge. This improvement is very large at higher reduced frequencies. The majority of the thrust is coming from near the leading edge of the airfoil. The optimum tension is a linear function with the tension equal to zero at the leading edge which allows the deflection to be larger there. The large deflection rotates the pressure difference normal at the surface toward the free stream direction. This provides a large contribution to thrust from the pressure difference across the airfoil which is added to the leading edge suction thrust from the plunging.

At low reduced frequencies the optimal spanwise tension function for maximum thrust is not zero at the leading edge. Further investigation shows that a corresponding change in the chordwise tension occurs at the same reduced frequency due to the existence of multiple maxima.

Conclusions

The aeroelastic equations of motion for a membrane wing undergoing prescribed rigid body motions are derived. The spanwise deformation was represented using a single shape reducing the membrane problem to a two dimensional form. The two dimensional airloads were determined from the aerodynamic theory presented in Walker and Patil\(^1\). Optimization of the aeroelastic system for one and three degrees of freedom was presented with divergence constraints. Results were presented for various reduced frequencies and mass ratios. It was shown that the stiffness of a flexible membrane wing undergoing unit harmonic plunging can be tailored to maximize stroke-averaged thrust. For all cases considered, there is a range of stiffness for which the flexible wing produces higher thrust as compared to rigid wing. Maximum thrust for a one, two, and three design variable system was presented. Increasing the design space by allowing the tension to be defined in a more general form always leads to a larger value of maximum thrust. The case of a linear variation of thrust yields an optimum that has the lowest spanwise tension at the leading edge. The results in the deformation which points the pressure vector more toward the free stream, increasing thrust. Larger values of mass ratio led to larger values of thrust at all reduced frequency for every design space investigated.

Investigation of systems with a larger design space is ongoing along with multi-objective optimization similar to the aerodynamic optimization presented in Walker, Patil and Canfield\(^2\). A constraint requiring the aeroelastic system to never go into flutter is also being considered. A global optimization technique is required for a solution to such problems and will be used in the future.

References


4Kaya, Mustafa and Tuncer, Ismail H. and Jones, Kevin D. and Platzer, Max F. Optimization of Aeroelastic Flapping Motion of Thin Airfoils in a Biplane Configuration for Maximum Thrust. 2007. 37th AIAA Fluid Dynamics Conference and Exhibit, Miami, Florida.

5Kaya, Mustafa and Tuncer, Ismail H. and Jones, Kevin D. and Platzer, Max F. Optimization of Flapping Motion of Airfoils in Biplane Configuration for Maximum Thrust and/or Efficiency. 2007. 37th AIAA Fluid Dynamics Conference and Exhibit, Miami, FL.


