ANALYSIS AND CONTROL OF NONLINEAR FLOW-STRUCTURE INTERACTIONS

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Abstract

We consider an aeroelastic model known as a nonlinear flow-structure interaction, which describes the oscillations of a thin, flexible plate immersed in a potential flow. This model is used to study the phenomenon known as flutter, which involves the coupling of the plate's vibrational modes with the aerodynamic load. Here, we treat this problem from a partial differential equations point of view and state results on well-posedness and long-time behavior of the system. The flow is described by a wave equation perturbed by a term which is proportional to the flow velocity. The plate is modeled by Kirchoff's equation; various boundary conditions and the physical (Berger and von Karman) nonlinearities are considered for the plate. Strong coupling between the two equations occurs in the acceleration potential (plate forcing term) and the down-wash (Neumann type boundary condition on the flow). The key parameters in this analysis correspond to the laminar flow velocity and the thickness of the plate.

Key Terms: aeroelasticity, flutter, fluid-structure interaction, nonlinear PDE, PDE control, long-time behavior, dynamical systems

1 Introduction

The interaction of a flexible structure with a surrounding flow of gas is a fundamental problem in aeroelasticity. There are a multitude of applications in engineering such as the stability of aircraft wings, bridges and buildings in response to a strong flow (or wind), and most large, flexible structures [16, 24, 3]. More recently, flutter has even been explored in the biological modeling of blood flow and snoring and sleep apnea, as well as in harvesting energy from fluttering objects.

We attempt to provide quantitative analysis and control of a model which arises in aeroelasticity and is governed by suitable PDE equations describing the interactive dynamics between an oscillating structure and a surrounding inviscid flow. We study solutions describing a flexible plate (e.g. an aircraft panel or thin wing) moving through the environment with a fixed velocity (subsonic or supersonic) and the corresponding flow of gas. One of the central problems in aeroelasticity is the determinantion of the speed of the aircraft corresponding to the onset of an enedemic instability termed wing "flutter" [14, 16, 17, 2, 3, 28]. Flutter is a self-sustaining instability which occurs when the elastic modes of the structure couple with the aerodynamic load due to the flow. It is well known that flutter in the wing of an aircraft may occur at high speeds, and can cause structural failure. Predicting the occurrence and magnitude of flutter, and determining the conditions which prevent or control such an instability are of prime concern in engineering and NASA applications.

The importance of this topic precipitates the great advances in the field. However, at present, most of the effort is experimental and computational. We refer to [14, 28, 15] for eloquent descriptions of such findings. While these methods address various aspects of the problem, they are based on finite-dimensional approximation of a continuum model fully described by partial differential equations (PDEs) [6, 7, 1, 15]. The PDE nature of the physical phenomena may not be adequately reflected by these approximations. This is particularly true when dealing with highly oscillatory models, where large frequencies, often causing instability, cannot accounted for.

While the ultimate control design must be physically feasible, and hence finite-dimensional, good understanding of infinite-dimensional phenomenon (described by PDEs) is fundamental for building effective approximations and finite-dimensional algorithms. Additionally, it is imperative that numerical studies are guided and checked against theoretical PDE predictions made directly from the mathematical model.

1.1 Outline

To begin a PDE study of the model, we must initially investigate well-posedness of the flow-structure interaction; that is the existence, uniqueness, and robustness of solutions arising in both *subsonic* and *supersonic* flows. Well-posedness for the flow-structure model is a necessary first step in the principal study of stability and control study of solutions.

2 Mathematical Model

2.1 PDE Model

The model in consideration involves the interaction of a clamped, partially clamped, or hinged plate with a field or flow of gas above it (we need only consider the flow on one side of the plate by antisymmetry). To describe the behavior of the gas, we make use of the theory of potential flows [6, 16, 17]. The oscillatory dynamics of the plate are governed by second-order nonlinear plate equations. For the plate, we consider a general class of 'physical' nonlinearities, which includes the von Karman and Berger nonlinearities of recent interest [13, 5]; these are suited for modeling plate dynamics with 'large' displacements, and therefore appropriate for flexible structures. Including nonlinearity in the structure of the model will turn out to be critical, not only for the sake of accuracy in modeling, but also because nonlinear effects play a principal role in stabilizing energies associated with the high frequency regime - the fundamental issue of our investigation.

The environment we consider is $\mathbb{R}^3_+ = \{(x,y,z) \in \mathbb{R}^3 \big| z \geq 0\}$. The thin plate has thickness $h \geq 0$ in the z-direction. The motion of the wing takes place in the negative x-direction at velocity U, with $U = 1$ corresponding to the speed of sound. (U can also be thought of as the unperturbed flow velocity in the x direction.) The top surface of the plate will be denoted $\Omega \subset \mathbb{R}^2$ and is assumed to be bounded in $\mathbb{R}^2$ with smooth boundary. The scalar function $\phi(x,y,z;t)$ gives potential flow. The scalar function $u(x,y;t)$ represents the z displacement of the central plane of the plate. Although the plate is 3-dimensional, we can view it in the context of flexible structures, and take it to be 2-dimensional, with negligible thickness (this is usual in the modeling of thin structures) [13]. This is tantamount to identifying the central plane of the plate with Ω, with the top surface of the plate where the flow-structure interaction takes place; this identification follows from the key assumption that the thickness $h << x-y$ span of the plate [18]. Hence, there is some ambiguity in taking the plate to have thickness, rather than actually assigning the plate a non-zero thickness. We address both cases, and their mathematical ramifications in remarks 2.1 and 2.3.

Our coupled system is as follows (taking $\mathbf{x} = (x,y,z)$ or (x,y), as dictated by context) and ν to be the outward normal direction to $\partial\Omega$:

First, the nonlinear Kirchoff plate equation with clamped boundary conditions $BC(u)$ is given by

$$\begin{cases} (1-\gamma\Delta)u_{tt}+\Delta^2u+f(u)=p(\mathbf{x},t) \text{ in } \Omega \\ u=\partial_\nu u=0 \text{ on } \partial\Omega \\ u(t=0)=u_0(\mathbf{x}),\ u_t(t=0)=u_1(\mathbf{x}). \end{cases} \tag{1}$$

Here $u_0,\ u_1$ are initial data. Other boundary conditions include: clamped, partially clamped, hinged (simply-supported), and free. These will be explicitly stated later in the text when used. By symmetry of the problem, we may assume that the flow of the gas occurs only above the plate in the x-direction, then $p(\mathbf{x},t)$ corresponds to the aerodynamical pressure of the flow on the plate and is given in terms of the flow $p(\mathbf{x},t)=(\partial_t+U\partial_x)\phi|_{z=0}$, $\mathbf{x}\in\Omega$, which is known as the 'accerlation potential'. The parameter $\gamma\geq 0$ represents rotational inertia in the filaments of the plate (the case $\gamma=0$ is said to be irrotational and occurs when the plate is taken to have zero thickness).

The nonlinearities we consider are the von Karman (f_V) and Berger (f_B) nonlinearities [5, 13, 4]. First, the von Karman nonlinearity is defined it in terms of Airy's stress function $v(\cdot)$ [11]:

$$f_V(u)\equiv -[v(u),u], \tag{2}$$

with $[\cdot,\cdot]$ denoting the von Karman bracket [13]. Secondly, for the Berger nonlinearity:

$$f_B(u)=-\Delta u\int_\Omega |u|^2. \tag{3}$$

These nonlinear terms are suitable for modeling the large oscillations of flexible structures (so-called large deflection theory). We are principally interested in the studying the von Karman nonlinearity, and in some sense, the Berger nonlinearity can be seen as its simplification. We will specify mathematical properties of these terms when they arise in later sections.

Secondly, a perturbed wave equation describes the flow:

$$\begin{cases} (\partial_t+U\partial_x)\phi=\Delta\phi \text{ in } \mathbb{R}^3_+ \\ \partial_z\phi=\begin{cases}(\partial_t+U\partial_x)u \text{ in } \Omega \\ 0 \text{ off } \Omega\end{cases} \end{cases} \tag{4}$$

The coupling between the plate and the flow occurs in the Neumann boundary condition, in the term known as a the 'down-wash'. The flow velocity U occurs here as a perturbation of flow equation.

Remark 2.1 *The two key parameters in our analysis are γ and U with regimes $\gamma=0$ (irrotational) or $\gamma>0$ (rotational), and $0\leq U<1$ (subsonic) or $U>1$ (supersonic). These parameters greatly affect the dynamics of the model, and many results listed below (and their proofs) depend critically on parameter regime in which we are working. The principal regime - the most interesting from the applied point of view, and most challenging mathematically - is taken to be $\gamma=0$, $U>1$.*

2.2 Energies

The energies for dynamical equations determine the state space and also which types of analysis will be necessary. These energies tend to arise from formal computations (using Green's Theorem) which will dictate the topological setup for our problem (the so-called 'finite-energy' considerations). Below, we list the plate energy $E_{pl}(t)$, the flow energy $E_{fl}(t)$, and the interactive energy $E_{int}(t)$. We will use the notation $\widehat{E}$ for a supersonic quantity, as some of the energies change (corresponding to a change of variable) in the supersonic case.

$$\begin{cases} E_{pl}(t)= & \frac{1}{2}\big\{||u_t||^2+||\Delta u||^2+\gamma||\nabla u_t||^2 \\ & +\Pi(u)\big\} \\ E_{fl}(t)= & \frac{1}{2}\big\{||\phi_t||^2+||\nabla\phi||^2-U^2||\partial_x\phi||^2\big\} \\ \widehat{E}_{fl}(t)= & \frac{1}{2}\big\{||(\partial_t+U\partial_x)\phi||^2+||\nabla\phi||^2\big\} \\ E_{int}(t)= & U<\partial_x u,\gamma[\phi]> \\ \widehat{E}_{int}(t)= & 0, \end{cases} \tag{5}$$

where $f(u) = \Pi'(u)$. We then have the total energies given by
$$\begin{cases}\mathcal{E}(t) = E_{pl}(t) + E_{fl}(t) + E_{int}(t) \\ \widehat{\mathcal{E}}(t) = E_{pl}(t) + \widehat{E}_{fl}(t).\end{cases}$$

Remark 2.2 (Notation) *above and for the remainder of the text, norms $||\cdot||$ are taken to be $L_2(D)$ for the domain dictated by context. Inner products in $L_2(\mathbb{R}^3_+)$ are written $(\cdot,\cdot)$, while inner products in $L_2(\Omega)$ are written $<\cdot,\cdot>$. Also, $H^s(D)$ will denote the Sobolev space of order s, defined on a domain D, and $H^s_0(D)$ denotes the closure of $C_0^\infty(D)$ in the $H^2(D)$ norm*

Each of the above terms (except $E_{int}(t)$) is positive and thus correspond to a physical energy. $E_{int}(t)$ has indeterminate sign, and hence in the subsonic case, $\mathcal{E}(t)$ is also indeterminate sign. This is one of the key issues to be dealt with in the case of subsonic well-posedness. In the subsonic case we have the (formal) energy balance law $\mathcal{E}(t) = \mathcal{E}(0)$; in the supersonic case we have $\widehat{\mathcal{E}}(t) + \int_0^t < u_t, (\partial_t + U\partial_x)\phi > dt = \widehat{\mathcal{E}}(0)$.

The aforementioned finite energy constraints manifest themselves in the natural requirements on the functions ϕ and u: $u \in C(0,T;H_0^2(\Omega)) \cap C^1(0,T;L^2(\Omega))$; $\phi \in C(0,T;H^1(\mathbb{R}^3_+)) \cap C^1(0,T;L^2(\mathbb{R}^3_+))$. Moreover, to set up the model in a dynamical systems framework, we will take our state space to be $Y = Y_{fl} + Y_{pl} \equiv \big(H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+)\big) \times \big(H_0^2(\Omega) \times L_2(\Omega)\big)$ for the state variable $y = (\phi, \psi; u, v)$. In the subsonic case we will consider the dynamical system with the state variable $y = (\phi, \phi_t; u, u_t)$, and in the supersonic case, the variable we consider is $(\phi, (\partial_t + U\partial_x)\phi; u, u_t)$.

Remark 2.3 *We pause here again to mention the mathematical issues pertaining to the parameters γ and U. In the case of $\gamma > 0$, the term $-\gamma\Delta u_{tt}$ is regularizing; it forces the plate velocity u_t to be in $H^1(\Omega)$ (changing the state space), as opposed to being in $L_2(\Omega)$ when $\gamma = 0$. This has major ramifications in the study of long-time behavior, as it corresponds to a loss of 'compactness' of the nonlinear term in the energy identity.*

Secondly, taking $U > 1$ requires a change of variable to produce a valid energy associated to the flow. This corresponds to the loss of strong ellipticity in the spatial flow operator, and leads to a degenerate static problem in semigroup considerations for the flow.

3 Mathematical Background

As alluded to above, we approach this problem for the PDE/dynamical systems point of view. To a certain extent, it is this approach which makes our results novel and more amenable to mathematical studies of long-time behavior and control. In this short section we present a basic framework, as well as terminology, in order to present the current results pertaining to the model above and provide context to the results.

3.1 Well-posedness

For a given PDE model, the primary consideration is that of Hadamard well-posedness. For a PDE or PDE system, well-posedness refers to the *existence, uniqueness, and continuous dependence on initial data* of finite energy solutions to the system for an arbitrary interval of time $[0,T]$. In this case, we refer to the semigroup well-posedness [25] of the system, namely the existence of a strongly continuous C_0 semigroup on the state space Y as described above. These notions of well-posedness and the existence of a C_0 semigroup can be thought to coincide in many cases of import. Here, for many of the results, we show the existence of a semigroup, which then directly or indirectly yields Hadamard well-posedness of the system. It is necessary when discussing well-posedness to a PDE or system of PDEs to specify what *type* of solution is being sought. In our case, finite energy solutions are identified with so called mild (or semigroup) solutions, which can again (often) be identified with *weak* solutions. In our exposition below, we are careful to specify which type of solutions pertain to a given result; however, for an explicit description of types of solutions for the model above in (1)-(4), please see [29].

These somewhat identical notions of well-posedness and the existence of a semigroup are important in applying dynamical systems considerations to the analysis of the model. Many techniques and powerful theorems are available for PDE systems which generate dynamical systems (evolutions, semigroups, and semiflows). In general, a **dynamical system** is a pair (X, S_t) where X is a separable metric space (typically Hilbert) and S_t is the so-called evolution operator; for each $t \in \mathbb{R}_+$, S_t is a continuous mapping from X to X, whose defining properties are the semigroup properties:

- $S_0 = Identity$, • $S_{t+s} = S_s \circ S_t$.

3.2 Long-time Behavior

Having defined a dynamical system, we will now discuss and define a few basic notions in dynamical systems theory which will be needed to state our results later in the text. The principal study of long-time behavior involves the stability of a system, specifically, the convergence of trajectories of the dynamical system to points of equilibria. This investigation is nontrivial, especially in the case of nonlinear dynamical systems.

First, a **dissipative** dynamical system (X, S_t) is a dynamical system with a uniform absorbing set D such that for any bounded set $B \subset X$ we have that $S_t B \subset D$ for t sufficiently large. The dynamical system is called **compact** if the absorbing set is compact in the state space X. (X, S_t) is said to be **asymptotically compact** if there exists a compact set $K \subset X$ which is uniformly attracting - i.e. for any bounded set B we have $\lim\limits_{t\to\infty} \text{dist}_X(S_t B; K) = 0$ where $\text{dist}_X(\cdot, \cdot)$ is the Hausdorff semi-distance. A dynamical system is said to be **asymptotically smooth** if for any bounded, forward invariant set D, there exists a compact set $K \subset \overline{D}$ which is uniformly attracting. Lastly, a **global attractor** is an invariant set which is uniformly attracting.

In this treatment we will discuss results in which attracting sets are shown to exist *plate component* of the model, i.e. for fixed flow data, we can show that there exists attracting sets which are uniform in the plate data. Additionally, the way we think of *asymptotic smoothness* is that an asymptotically smooth dynamical system has *local compact attracting sets* - i.e. for a given bounded, invariant (with respect to the dynamics) set B_R, there exists a compact attracting set K for B_R (which depends on B_R).

We are interested in showing that our (nonlinear) dynamical system has a global attracting set with respect to the plate dynamics. So, although the dynamical system generated pertains to the entire four variable state space for the plate and flow, the attracting set will pertain only to the plate component. This type of result is physical, and 'optimal', as we must take the flow data to be given (in this model, we have no control over the initial state of the ambient atmosphere). We do implement results on global attractors, but we do so as we reduce the flow to a retarded plate term, thereby suppressing the dependence of the attractor on the initial flow data.

After showing the existence of an attracting set for the plate, often the next step is to show that this set has finite dimension. Ultimately, this is tantamount to reducing the infinite dimensional dynamics of a PDE system to a finite dimensional set in the state space, to which classical, finite dimensional control (stability) theory can be applied.

4 Previous Results

Although there is a wealth of results pertaining to flow-structure models, infinite-dimensional analyses based on quantitative studies of PDEs are scarce. Issues such as global existence and uniqueness of PDE solutions have only recently been addressed in detail. This is not surprising, in view of fundamental mathematical difficulties associated with the coupling of two PDEs at an interface. Many existing results pertain to models that are highly *regularized*. These models contain regularizing terms or including strong [7, 8] or thermal damping [26, 27] into the model) which make the structural equations parabolic-like. Thus, the underlying analysis is simpler, due to the regularity inherited from diffusive parabolic effects.

Additionally, many of the techniques and strategies invoked are drawn from studies of long-time nonlinear plate dynamics; particularly, the study of the existence of global compact attractors and determination of their dimension, geometric structure, and smoothness properties. These results are collected and nicely presented in [10, 11].

We now discuss the particular results which had been established for the flow-structure model given in (1)-(4) prior to the PI (and coauthors') current project. In what follows, *well-posedness* is to mean: for all T, (1)-(4) has unique semigroup (and weak) solutions on $[0, T]$ for initial data in the state space. If we assume the initial data has additional regularity (and the obvious compatibility conditions holds) then we have strong (i.e. classical) solutions.

4.1 Well-posedness

Theorem 4.1 **($U \neq 1,\ \gamma > 0$)** *Take the system in (1)-(4) with clamped boundary conditions for the plate. Assume the initial data* $(\phi_0, \phi_1; u_0, u_1) \in H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times H^2_0(\Omega) \times L_2(\Omega)$. *We may take f to be any nonlinearity which is locally Lipschitz from $H^2(\Omega) \to L_2(\Omega)$ (in particular, $f = f_V$ or f_B). Then, the system is well-posed.*

The proof of the above theorem is not uniform in the parameter γ. Moreover, multiple methods of proof are available. We cite [11] and the references therein, which gives a technical formulation of the well-posedness problem and discusses the various approaches (Galerkin and viscosity) of showing well-posedness.

Theorem 4.2 **($0 \leq U < 1,\ \gamma = 0$)** *Take the system in (1)-(4) with clamped boundary conditions for the plate. Assume the initial data* $(\phi_0, \phi_1; u_0, u_1) \in H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times H^2_0(\Omega) \times L_2(\Omega)$. *Take $f = f_V$. Then, the system is well-posed.*

We again cite [11] for a nice presentation of well-posedness in this case. One key issue with well-posedness, as proven in the aforementioned reference, is that semigroup methods are not utilized. This, ultimately, yields well-posedness, but also estimates which are sensitive to the parameter γ. An ideal well-posedness result (in this case) would yield estimates which are independent of γ, making the analysis more amenable to long-time behavior studies.

Remark 4.1 *The proof in the above case, as well as the long-time behavior results which are presented in the section immediately following this remark, are critically dependent upon a reduction result in [9] (and restated in [11]) which reduces the flow component of the system, i.e. the flow can be written as a retarded potential of the plate, which allows us to consider the (closed) plate system as a retarded PDE (without coupling). The formula which allows this reduction is highly important in the long-time behavior results obtained in this project as well.*

4.2 Long-time Behavior

The theorem in this section actually pertains to a modified model: in particular, the available long-time behavior result requires the addition of a term $k(1 - \gamma\Delta)\partial_t u$ to the LHS of (1).

Theorem 4.3 *Take the system in (1)-(4) with the aforementioned modification to the LHS of the plate equation. Also, take clamped boundary conditions for the plate, and assume that the flow data ϕ_0 and ϕ_1 has compact support in $\mathbb{R}^3_+$. Take $\gamma > 0$ and $U \geq 0,\ U \neq 1$. Denote the weak (or mild)* *solution $(\phi, \phi_t; u, u_t) \in H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times H^2_0(\Omega) \times L_2(\Omega)$. Then there exists an attracting compact set $\mathscr{U} \subset H^2_0(\Omega) \times L_2(\Omega)$ of finite fractal dimension for the plate component of the state space, which is uniformly attracting for (u, u_t).*

This theorem's proof can again be found in [11].

We again emphasize that the attracting set is uniform with respect to plate data only, and this will be the case with all later results on 'attractors' for this system. This result is interesting since it applies to all flow velocities $U \neq 1$, i.e. the method of proof does not break down crossing the threshold $U = 1$. However, to obtain this result, strong damping in the form of the term $(1 - \gamma\Delta)\partial_t u$ is necessary.

5 Current Results

The results listed above bring us to the current state of research for the nonlinear flow-structure model

5.1 Well-posedness

The first well-posedness result concerns the subsonic case. In particular, we address all parameter values $\gamma \geq 0$ simultaneously.

Theorem 5.1 $(0 \leq U < 1,\ \gamma \geq 0)$ *Take the system in (1)-(4) with clamped boundary conditions for the plate. Assume the initial data $(\phi_0, \phi_1; u_0, u_1) \in H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times H^2_0(\Omega) \times L_2(\Omega)$. We may take f to be any nonlinearity which is locally Lipschitz from $H^2(\Omega) \to L_2(\Omega)$ (in particular, $f = f_V$ or f_B). Then, the system is well-posed.*

Remark 5.1 *The above theorem also holds for clamped, hinged, and free boundary conditions on the plate.*

The proof of the above theorem makes use of semigroup theory, and provides a general framework for using m-dissipative operators on non-dissipative problems. The details are too technical for this treatment, but suffice it to say that the primary contribution of this treatment [29] lies in the proof of this theorem and the semigroup estimates which it provides. This theorem (and proof) also unifies the treatment of the cases $\gamma = 0$ and $\gamma > 0$, and provides estimates on the solution which are independent of γ, and the solutions are bounded as $t \to \infty$.

The second theorem we obtained deals with subsonic well-posedness in the case of hinged boundary damping. We introduce a boundary damping mechanism which is intended to help in stabilizing solutions. Instead of taking clamped boundary conditions for the plate, we replace the boundary conditions of (1) with

$$u = 0,\ \Delta u = -g(\partial_\nu u_t) \text{ on } \partial\Omega,$$

where g is a continuous, monotone increasing function. Before considering long-time behavior of solutions with this boundary control mechanism, we must first show well-posedness of the system with nonlinear boundary damping. For notation, denote $L_2^\gamma \equiv \begin{cases} H^1(\Omega) & \gamma > 0 \\ L_2(\Omega) & \gamma = 0 \end{cases}$.

Theorem 5.2 *Take the system in (1)-(4) with the aforementioned hinged dissipation boundary conditions for the plate. Assume the initial data $(\phi_0, \phi_1; u_0, u_1) \in H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times (H^2 \cap H^1_0)(\Omega) \times L_2^\gamma(\Omega)$. Take f to be either f_V or f_B. Then, the system is well-posed. Moreover, the nonlinear semigroup which generates the solution is bounded, with an operator bound which does not depend on γ or $t > 0$.*

The above theorem gives well-posedness of the model in the presence of nonlinear boundary damping, which leads to long-time behavior considerations. The proof is found in [22], and this paper has additional results on the regularity of such solutions and the strong convergence of the generated nonlinear semigroups $S_\gamma(t) \to S_0(t)$ on any fixed interval $[0, T]$; this result demonstrates the

the nonlinear dynamics of the coupled nonlinear flow-structure interaction in the case $\gamma = 0$ can be viewed as the limiting case of the more regular dynamics in the case of non-neglibile plate thickness, $\gamma > 0$.

Lastly (and most recently), the issue of supersonic well-posedness has been addressed (the case $\gamma \geq 0, \; U > 1$). This is perhaps the most exciting (and difficult to obtain) result in the project to date. Moreover, this is the most desirable result from the industrial and applied point of view. Demonstrating well-posedness in this case (specifically using semigroup methods) has been a foremost goal of many of the authors who have worked on this model. We have obtained well-posedness (via two different proof methods), however, these results have yet to be published. They are to appear in the forthcoming manuscript [12].

Theorem 5.3 **($U > 1, \; \gamma \geq 0$)** *Take (1)-(4) with clamped boundary conditions. Additionally, assume $U > 1$. Then the system is well-posed.*

Remark 5.2 *Two important aspects of this result are: (a) it fundamentally depends on the plate's boundary conditions (at one point in the proof plate solutions must be extended to the whole $x - y$ plane and this cannot be done for arbitrary boundary conditions); the proof only obtains in the case of clamped or hinged boundary conditions (at least as currently written); secondly (b), the semigroup obtained in this case is not bounded (unlike the subsonic case). This has to do with the change of state variable ($\phi_t \to \phi_t + U\phi_x$) which must be made in performing the semigroup analysis, and unlike in the previous case, the flow energies cannot be controlled.*

5.2 Long-time Behavior

The current state of research for long-time behavior of the nonlinear flow-structure interaction is relegated to the subsonic case $0 \leq U < 1$. Also, we restrict our attention to the case $\gamma = 0$, which is typically the most demanding mathematically; the hope is to then extend the method of proof to the case for all $\gamma \geq 0$. Hence, in this section all results take $0 \leq U < 1$ and $\gamma = 0$. Here we implement the nonlinear boundary damping, as mentioned above, and investigate the existence of global attracting sets. It will be necessary in this section to differentiate the results based on which nonlinearity is being considered.

Theorem 5.4 (Asymptotic Smoothness) *Consider the system in (1)-(4) with boundary conditions for the plate given by $u = 0$, $\Delta u = -g(\partial_\nu u_t)$ on $\partial\Omega$, where g is a continuous, monotone increasing function. Take $f = f_V$ or f_B. By our well-posedness result, the model generates a dynamical system on the state space $H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+) \times (H^2 \cap H^1_0)(\Omega) \times L_2(\Omega)$. Then the dynamical system generated by(1)-(4) is asymptotically smooth, and hence has local compact attracting sets for semigroup (and thus weak) solutions to the plate component of the solution: (u, u_t).*

In the case when $f = f_B$, we may exploit the structure of the nonlinearity, and impose a mild geometric condition on the plate, to show that the dynamical system is dissipative, and hence making use a well-known result in dynamical systems theory, we have:

Theorem 5.5 *Under the same assumptions of the previous theorem, take $f = f_B$ and assume the domain Ω is star-shaped. Also, assume that the flow data $(\phi_0, \phi_1) \in H^1(\mathbb{R}^3_+) \times L_2(\mathbb{R}^3_+)$ is compactly supported. Then there exists a unique compact attracting set $\mathscr{U}$ for the plate component of the dynamical system, (u, u_t), which is uniform with respect to initial plate data.*

Remark 5.3 *Although the above asymptotic smoothness result is nontrivial, and produces local compact attracting sets, the desired result is of course to show that a unique compact attracting set exists for $f = f_V$. To date, this has not been possible with the boundary damping in the previous theorem; only by exploiting the special structure* *of f_B and making use of the geometric condition on Ω were we able to show this. This leads us to consider other types of possible damping, as discussed in the following section.*

6 Conclusions and Open Questions

To date, we have made substantial progress in the problem of control of nonlinear flow-structure interactions. In particular, we have shown in all cases (in a quasi-unified semigroup treatment), the well-posedness of the model. This leads us to long-time behavior considerations. In the subsonic case, we have investigated a particular type of boundary damping, which produced results, but did not provide fully provide the desired result in the case of the von Karman flow-structure interaction.

We now briefly list the research topics which remain open, to some extent:

1. Existence of attractors for the von Karman flow-structure interaction. As mentioned above, nonlinear boundary damping was not sufficient to provide the desired result on the existence of global compact attractors in this case. In the subsonic case $U < 1$, we need a stronger damping mechanism in order to continue this avenue of research. The next type of damping to consider is *nonlinear interior damping localized near the boundary*. This type of damping is, in some sense, more physical than pure boundary damping. However, before this damping could be considered in the context of flow-structure problems, it needed to be investigated in the case of the von Karman plate alone (independent of the flow coupling). This was done during the fall of 2011, and will appear in the forthcoming manuscript [20]. Upon considering this damping, we are confident that the dynamics of the von Karman flow-structure interaction will yield a compact global attractor, and we may then investigate its dimensionality and regularity properties.

2. Properties of solutions. To date, we have only investigated certain regularity properties of solutions in the paper [22]. Future studies will address regularity (perhaps optimal) of all semigroup solutions obtained to date. These problems are tantamount to characterizing the domain of the generator for the dynamics, and are technical but beneficial investigations.

3. Supersonic well-posedness. It is worthwhile to investigate well-posedness for the case $U > 1$ for all standard plate boundary conditions. Moreover, we must investigate well-posedness in the presence of some damping mechanism which will force the dynamics to be bounded. This leads to the next bullet:

4. Supersonic long-time behavior. We must first find an appropriate damping mechanism which will produce boundedness (or decay) of the solutions on the finite energy space. Then, after showing well-posedness of the model with this control mechanism, we must begin to investigate existence and properties of global attractors.

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